

## UNITS, PHYSICAL QUANTITIES AND VECTORS

- 1.1. IDENTIFY:** Convert units from mi to km and from km to ft.

**SET UP:** 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.

**EXECUTE:** (a)  $1.00 \text{ mi} = (1.00 \text{ mi}) \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) = 1.61 \text{ km}$

(b)  $1.00 \text{ km} = (1.00 \text{ km}) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right) \left( \frac{10^2 \text{ cm}}{1 \text{ m}} \right) \left( \frac{1 \text{ in.}}{2.54 \text{ cm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) = 3.28 \times 10^3 \text{ ft}$

**EVALUATE:** A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

- 1.2. IDENTIFY:** Convert volume units from L to in.<sup>3</sup>.

**SET UP:** 1 L = 1000 cm<sup>3</sup>. 1 in. = 2.54 cm

**EXECUTE:**  $0.473 \text{ L} \times \left( \frac{1000 \text{ cm}^3}{1 \text{ L}} \right) \times \left( \frac{1 \text{ in.}}{2.54 \text{ cm}} \right)^3 = 28.9 \text{ in.}^3$

**EVALUATE:** 1 in.<sup>3</sup> is greater than 1 cm<sup>3</sup>, so the volume in in.<sup>3</sup> is a smaller number than the volume in cm<sup>3</sup>, which is 473 cm<sup>3</sup>.

- 1.3. IDENTIFY:** We know the speed of light in m/s.  $t = d/v$ . Convert 1.00 ft to m and  $t$  from s to ns.

**SET UP:** The speed of light is  $v = 3.00 \times 10^8 \text{ m/s}$ . 1 ft = 0.3048 m. 1 s = 10<sup>9</sup> ns.

**EXECUTE:**  $t = \frac{0.3048 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.02 \times 10^{-9} \text{ s} = 1.02 \text{ ns}$

**EVALUATE:** In 1.00 s light travels  $3.00 \times 10^8 \text{ m} = 3.00 \times 10^5 \text{ km} = 1.86 \times 10^5 \text{ mi}$ .

- 1.4. IDENTIFY:** Convert the units from g to kg and from cm<sup>3</sup> to m<sup>3</sup>.

**SET UP:** 1 kg = 1000 g. 1 m = 1000 cm.

**EXECUTE:**  $11.3 \frac{\text{g}}{\text{cm}^3} \times \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) \times \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 1.13 \times 10^4 \frac{\text{kg}}{\text{m}^3}$

**EVALUATE:** The ratio that converts cm to m is cubed, because we need to convert cm<sup>3</sup> to m<sup>3</sup>.

- 1.5. IDENTIFY:** Convert volume units from in.<sup>3</sup> to L.

**SET UP:** 1 L = 1000 cm<sup>3</sup>. 1 in. = 2.54 cm.

**EXECUTE:**  $(327 \text{ in.}^3) \times (2.54 \text{ cm/in.})^3 \times (1 \text{ L}/1000 \text{ cm}^3) = 5.36 \text{ L}$

**EVALUATE:** The volume is 5360 cm<sup>3</sup>. 1 cm<sup>3</sup> is less than 1 in.<sup>3</sup>, so the volume in cm<sup>3</sup> is a larger number than the volume in in.<sup>3</sup>.

- 1.6. IDENTIFY:** Convert ft<sup>2</sup> to m<sup>2</sup> and then to hectares.

**SET UP:** 1.00 hectare = 1.00 × 10<sup>4</sup> m<sup>2</sup>. 1 ft = 0.3048 m.

**EXECUTE:** The area is  $(12.0 \text{ acres}) \left( \frac{43,600 \text{ ft}^2}{1 \text{ acre}} \right) \left( \frac{0.3048 \text{ m}}{1.00 \text{ ft}} \right)^2 \left( \frac{1.00 \text{ hectare}}{1.00 \times 10^4 \text{ m}^2} \right) = 4.86 \text{ hectares}$ .

**EVALUATE:** Since 1 ft = 0.3048 m, 1 ft<sup>2</sup> = (0.3048)<sup>2</sup> m<sup>2</sup>.

- 1.7. IDENTIFY:** Convert seconds to years.

**SET UP:** 1 billion seconds = 1 × 10<sup>9</sup> s. 1 day = 24 h. 1 h = 3600 s.

**EXECUTE:**  $1.00 \text{ billion seconds} = (1.00 \times 10^9 \text{ s}) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right) \left( \frac{1 \text{ y}}{365 \text{ days}} \right) = 31.7 \text{ y}$ .

**EVALUATE:** The conversion  $1 \text{ y} = 3.156 \times 10^7 \text{ s}$  assumes  $1 \text{ y} = 365.24 \text{ d}$ , which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

**1.8. IDENTIFY:** Apply the given conversion factors.

**SET UP:** 1 furlong = 0.1250 mi and 1 fortnight = 14 days. 1 day = 24 h.

**EXECUTE:**  $(180,000 \text{ furlongs/fortnight}) \left( \frac{0.125 \text{ mi}}{1 \text{ furlong}} \right) \left( \frac{1 \text{ fortnight}}{14 \text{ days}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right) = 67 \text{ mi/h}$

**EVALUATE:** A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

**1.9. IDENTIFY:** Convert miles/gallon to km/L.

**SET UP:** 1 mi = 1.609 km. 1 gallon = 3.788 L.

**EXECUTE:** (a)  $55.0 \text{ miles/gallon} = (55.0 \text{ miles/gallon}) \left( \frac{1.609 \text{ km}}{1 \text{ mi}} \right) \left( \frac{1 \text{ gallon}}{3.788 \text{ L}} \right) = 23.4 \text{ km/L}$ .

(b) The volume of gas required is  $\frac{1500 \text{ km}}{23.4 \text{ km/L}} = 64.1 \text{ L}$ .  $\frac{64.1 \text{ L}}{45 \text{ L/tank}} = 1.4 \text{ tanks}$ .

**EVALUATE:** 1 mi/gal = 0.425 km/L. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so 1 mi/gal  $\sim \frac{1}{4}$  km/L, which is roughly our result.

**1.10. IDENTIFY:** Convert units.

**SET UP:** Use the unit conversions given in the problem. Also, 100 cm = 1 m and 1000 g = 1 kg.

**EXECUTE:** (a)  $\left( 60 \frac{\text{mi}}{\text{h}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 88 \frac{\text{ft}}{\text{s}}$

(b)  $\left( 32 \frac{\text{ft}}{\text{s}^2} \right) \left( \frac{30.48 \text{ cm}}{1 \text{ ft}} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) = 9.8 \frac{\text{m}}{\text{s}^2}$

(c)  $\left( 1.0 \frac{\text{g}}{\text{cm}^3} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 \left( \frac{1 \text{ kg}}{1000 \text{ g}} \right) = 10^3 \frac{\text{kg}}{\text{m}^3}$

**EVALUATE:** The relations  $60 \text{ mi/h} = 88 \text{ ft/s}$  and  $1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$  are exact. The relation  $32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$  is accurate to only two significant figures.

**1.11. IDENTIFY:** We know the density and mass; thus we can find the volume using the relation

density = mass/volume =  $m/V$ . The radius is then found from the volume equation for a sphere and the result for the volume.

**SET UP:** Density =  $19.5 \text{ g/cm}^3$  and  $m_{\text{critical}} = 60.0 \text{ kg}$ . For a sphere  $V = \frac{4}{3}\pi r^3$ .

**EXECUTE:**  $V = m_{\text{critical}} / \text{density} = \left( \frac{60.0 \text{ kg}}{19.5 \text{ g/cm}^3} \right) \left( \frac{1000 \text{ g}}{1.0 \text{ kg}} \right) = 3080 \text{ cm}^3$ .

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080 \text{ cm}^3)} = 9.0 \text{ cm}.$$

**EVALUATE:** The density is very large, so the 130 pound sphere is small in size.

**1.12. IDENTIFY:** Use your calculator to display  $\pi \times 10^7$ . Compare that number to the number of seconds in a year.

**SET UP:** 1 yr = 365.24 days, 1 day = 24 h, and 1 h = 3600 s.

**EXECUTE:**  $(365.24 \text{ days/1 yr}) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 3.15567 \dots \times 10^7 \text{ s}$ ;  $\pi \times 10^7 \text{ s} = 3.14159 \dots \times 10^7 \text{ s}$

The approximate expression is accurate to two significant figures.

**EVALUATE:** The close agreement is a numerical accident.

**1.13. IDENTIFY:** The percent error is the error divided by the quantity.

**SET UP:** The distance from Berlin to Paris is given to the nearest 10 km.

**EXECUTE:** (a)  $\frac{10 \text{ m}}{890 \times 10^3 \text{ m}} = 1.1 \times 10^{-3}\%$ .

(b) Since the distance was given as 890 km, the total distance should be 890,000 meters. We know the total distance to only three significant figures.

**EVALUATE:** In this case a very small percentage error has disastrous consequences.

**1.14. IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

**SET UP:** 12 mm has two significant figures and 5.98 mm has three significant figures.

**EXECUTE:** (a)  $(12 \text{ mm}) \times (5.98 \text{ mm}) = 72 \text{ mm}^2$  (two significant figures)

(b)  $\frac{5.98 \text{ mm}}{12 \text{ mm}} = 0.50$  (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

**EVALUATE:** The length of the rectangle is known only to the nearest mm, so the answers in parts (c) and (d) are known only to the nearest mm.

**1.15. IDENTIFY and SET UP:** In each case, estimate the precision of the measurement.

**EXECUTE:** (a) If a meter stick can measure to the nearest millimeter, the error will be about 0.13%.

(b) If the chemical balance can measure to the nearest milligram, the error will be about  $8.3 \times 10^{-3}\%$ .

(c) If a handheld stopwatch (as opposed to electric timing devices) can measure to the nearest tenth of a second, the error will be about  $2.8 \times 10^{-2}\%$ .

**EVALUATE:** The percent errors are those due only to the limit of precision of the measurement.

**1.16. IDENTIFY:** Use the extreme values in the piece's length and width to find the uncertainty in the area.

**SET UP:** The length could be as large as 5.11 cm and the width could be as large as 1.91 cm.

**EXECUTE:** The area is  $9.69 \pm 0.07 \text{ cm}^2$ . The fractional uncertainty in the area is  $\frac{0.07 \text{ cm}^2}{9.69 \text{ cm}^2} = 0.72\%$ , and the

fractional uncertainties in the length and width are  $\frac{0.01 \text{ cm}}{5.10 \text{ cm}} = 0.20\%$  and  $\frac{0.01 \text{ cm}}{1.9 \text{ cm}} = 0.53\%$ . The sum of these

fractional uncertainties is  $0.20\% + 0.53\% = 0.73\%$ , in agreement with the fractional uncertainty in the area.

**EVALUATE:** The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

**1.17. IDENTIFY:** Calculate the average volume and diameter and the uncertainty in these quantities.

**SET UP:** Using the extreme values of the input data gives us the largest and smallest values of the target variables and from these we get the uncertainty.

**EXECUTE:** (a) The volume of a disk of diameter  $d$  and thickness  $t$  is  $V = \pi(d/2)^2 t$ .

The average volume is  $V = \pi(8.50 \text{ cm}/2)^2(0.50 \text{ cm}) = 2.837 \text{ cm}^3$ . But  $t$  is given to only two significant figures so the answer should be expressed to two significant figures:  $V = 2.8 \text{ cm}^3$ .

We can find the uncertainty in the volume as follows. The volume could be as large as

$V = \pi(8.52 \text{ cm}/2)^2(0.055 \text{ cm}) = 3.1 \text{ cm}^3$ , which is  $0.3 \text{ cm}^3$  larger than the average value. The volume could be as small as  $V = \pi(8.52 \text{ cm}/2)^2(0.045 \text{ cm}) = 2.5 \text{ cm}^3$ , which is  $0.3 \text{ cm}^3$  smaller than the average value. The

uncertainty is  $\pm 0.3 \text{ cm}^3$ , and we express the volume as  $V = 2.8 \pm 0.3 \text{ cm}^3$ .

(b) The ratio of the average diameter to the average thickness is  $8.50 \text{ cm}/0.050 \text{ cm} = 170$ . By taking the largest possible value of the diameter and the smallest possible thickness we get the largest possible value for this ratio:  $8.52 \text{ cm}/0.045 \text{ cm} = 190$ . The smallest possible value of the ratio is  $8.48/0.055 = 150$ . Thus the uncertainty is  $\pm 20$  and we write the ratio as  $170 \pm 20$ .

**EVALUATE:** The thickness is uncertain by 10% and the percentage uncertainty in the diameter is much less, so the percentage uncertainty in the volume and in the ratio should be about 10%.

**1.18. IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

**SET UP:** Estimate  $3 \times 10^8$  people, so  $2 \times 10^8$  cars.

**EXECUTE:**  $(\text{Number of cars} \times \text{miles/car day}) / (\text{mi/gal}) = \text{gallons/day}$

$$(2 \times 10^8 \text{ cars} \times 10000 \text{ mi/yr/car} \times 1 \text{ yr}/365 \text{ days}) / (20 \text{ mi/gal}) = 3 \times 10^8 \text{ gal/day}$$

**EVALUATE:** The number of gallons of gas used each day approximately equals the population of the U.S.

**1.19. IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

**SET UP:** A mass of 1 kg is equivalent to a weight of about 2.2 lbs.  $1 \text{ in.} = 2.54 \text{ cm}$ .  $1 \text{ y} = 12 \text{ months}$ .

**EXECUTE:** (a) 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

(b)  $200 \text{ m} = (2.00 \times 10^4 \text{ cm}) \left( \frac{1 \text{ in.}}{2.54 \text{ cm}} \right) = 7.9 \times 10^3 \text{ inches}$ . This is much greater than the height of a person.

(c)  $200 \text{ cm} = 2.00 \text{ m} = 79 \text{ inches} = 6.6 \text{ ft}$ . Some people are this tall, but not an ordinary man.

(d) 200 mm = 0.200 m = 7.9 inches . This is much too short.

(e) 200 months =  $(200 \text{ mon}) \left( \frac{1 \text{ y}}{12 \text{ mon}} \right) = 17 \text{ y}$  . This is the age of a teenager; a middle-aged man is much older than this.

**EVALUATE:** None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

**1.20. IDENTIFY:** The number of kernels can be calculated as  $N = V_{\text{bottle}} / V_{\text{kernel}}$  .

**SET UP:** Based on an Internet search, Iowan corn farmers use a sieve having a hole size of 0.3125 in.  $\approx 8$  mm to remove kernel fragments. Therefore estimate the average kernel length as 10 mm, the width as 6 mm and the depth as 3 mm. We must also apply the conversion factors 1 L = 1000 cm<sup>3</sup> and 1 cm = 10 mm.

**EXECUTE:** The volume of the kernel is:  $V_{\text{kernel}} = (10 \text{ mm})(6 \text{ mm})(3 \text{ mm}) = 180 \text{ mm}^3$  . The bottle's volume is:

$$V_{\text{bottle}} = (2.0 \text{ L}) \left[ \frac{(1000 \text{ cm}^3)}{(1.0 \text{ L})} \right] \left[ \frac{(10 \text{ mm})^3}{(1.0 \text{ cm})^3} \right] = 2.0 \times 10^6 \text{ mm}^3 .$$

$$N_{\text{kernels}} = V_{\text{bottle}} / V_{\text{kernels}} \approx (2.0 \times 10^6 \text{ mm}^3) / (180 \text{ mm}^3) = 11,000 \text{ kernels} .$$

**EVALUATE:** This estimate is highly dependent upon your estimate of the kernel dimensions. And since these dimensions vary amongst the different available types of corn, acceptable answers could range from 6,500 to 20,000.

**1.21. IDENTIFY:** Estimate the number of pages and the number of words per page.

**SET UP:** Assuming the two-volume edition, there are approximately a thousand pages, and each page has between 500 and a thousand words (counting captions and the smaller print, such as the end-of-chapter exercises and problems).

**EXECUTE:** An estimate for the number of words is about  $10^6$  .

**EVALUATE:** We can expect that this estimate is accurate to within a factor of 10.

**1.22. IDENTIFY:** Approximate the number of breaths per minute. Convert minutes to years and cm<sup>3</sup> to m<sup>3</sup> to find the volume in m<sup>3</sup> breathed in a year.

**SET UP:** Assume 10 breaths/min .  $1 \text{ y} = (365 \text{ d}) \left( \frac{24 \text{ h}}{1 \text{ d}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = 5.3 \times 10^5 \text{ min}$  .  $10^2 \text{ cm} = 1 \text{ m}$  so

$10^6 \text{ cm}^3 = 1 \text{ m}^3$  . The volume of a sphere is  $V = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3$  , where  $r$  is the radius and  $d$  is the diameter. Don't forget to account for four astronauts.

**EXECUTE:** (a) The volume is  $(4)(10 \text{ breaths/min})(500 \times 10^{-6} \text{ m}^3) \left( \frac{5.3 \times 10^5 \text{ min}}{1 \text{ y}} \right) = 1 \times 10^4 \text{ m}^3 / \text{yr}$  .

$$(b) d = \left( \frac{6V}{\pi} \right)^{1/3} = \left( \frac{6[1 \times 10^4 \text{ m}^3]}{\pi} \right)^{1/3} = 27 \text{ m}$$

**EVALUATE:** Our estimate assumes that each cm<sup>3</sup> of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

**1.23. IDENTIFY:** Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

**SET UP:** Estimate that we blink 10 times per minute.  $1 \text{ y} = 365 \text{ days}$  .  $1 \text{ day} = 24 \text{ h}$  ,  $1 \text{ h} = 60 \text{ min}$  . Use 80 years for the lifetime.

**EXECUTE:** The number of blinks is  $(10 \text{ per min}) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{365 \text{ days}}{1 \text{ y}} \right) (80 \text{ y/lifetime}) = 4 \times 10^8$

**EVALUATE:** Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

**1.24. IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

**SET UP:** An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

**EXECUTE:**  $N_{\text{beats}} = (75 \text{ beats/min}) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{365 \text{ days}}{\text{yr}} \right) \left( \frac{80 \text{ yr}}{\text{lifespan}} \right) = 3 \times 10^9 \text{ beats/lifespan}$

$$V_{\text{blood}} = (50 \text{ cm}^3 / \text{beat}) \left( \frac{1 \text{ L}}{1000 \text{ cm}^3} \right) \left( \frac{1 \text{ gal}}{3.788 \text{ L}} \right) \left( \frac{3 \times 10^9 \text{ beats}}{\text{lifespan}} \right) = 4 \times 10^7 \text{ gal/lifespan}$$

**EVALUATE:** This is a very large volume.

**1.25. IDENTIFY:** Estimation problem

**SET UP:** Estimate that the pile is 18 in.  $\times$  18 in.  $\times$  5 ft 8 in.. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

**EXECUTE:** The volume of gold in the pile is  $V = 18 \text{ in.} \times 18 \text{ in.} \times 68 \text{ in.} = 22,000 \text{ in.}^3$ . Convert to  $\text{cm}^3$ :

$$V = 22,000 \text{ in.}^3 (1000 \text{ cm}^3 / 61.02 \text{ in.}^3) = 3.6 \times 10^5 \text{ cm}^3.$$

The density of gold is  $19.3 \text{ g/cm}^3$ , so the mass of this volume of gold is

$$m = (19.3 \text{ g/cm}^3)(3.6 \times 10^5 \text{ cm}^3) = 7 \times 10^6 \text{ g}.$$

The monetary value of one gram is \$10, so the gold has a value of  $(\$10/\text{gram})(7 \times 10^6 \text{ grams}) = \$7 \times 10^7$ , or about  $\$100 \times 10^6$  (one hundred million dollars).

**EVALUATE:** This is quite a large pile of gold, so such a large monetary value is reasonable.

**1.26. IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in  $\text{m}^3$ . Convert  $\text{m}^3$  to L.

**SET UP:** Estimate the diameter of a drop to be  $d = 2 \text{ mm}$ . The volume of a spherical drop is  $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$ .  $10^3 \text{ cm}^3 = 1 \text{ L}$ .

**EXECUTE:**  $V = \frac{1}{6}\pi(0.2 \text{ cm})^3 = 4 \times 10^{-3} \text{ cm}^3$ . The number of drops in 1.0 L is  $\frac{1000 \text{ cm}^3}{4 \times 10^{-3} \text{ cm}^3} = 2 \times 10^5$

**EVALUATE:** Since  $V \sim d^3$ , if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

**1.27. IDENTIFY:** Estimate the number of students and the average number of pizzas eaten by each student in a school year.

**SET UP:** Assume a school of thousand students, each of whom averages ten pizzas a year (perhaps an underestimate)

**EXECUTE:** They eat a total of  $10^4$  pizzas.

**EVALUATE:** The same answer applies to a school of 250 students averaging 40 pizzas a year each.

**1.28. IDENTIFY:** The number of bills is the distance to the moon divided by the thickness of one bill.

**SET UP:** Estimate the thickness of a dollar bills by measuring a short stack, say ten, and dividing the measurement by the total number of bills. I obtain a thickness of roughly 1 mm. From Appendix F, the distance from the earth to the moon is  $3.8 \times 10^8 \text{ m}$ .

**EXECUTE:**  $N_{\text{bills}} = \left( \frac{3.8 \times 10^8 \text{ m}}{0.1 \text{ mm/bill}} \right) \left( \frac{10^3 \text{ mm}}{1 \text{ m}} \right) = 3.8 \times 10^{12} \text{ bills} \approx 4 \times 10^{12} \text{ bills}$

**EVALUATE:** This answer represents 4 trillion dollars! The cost of a single space shuttle mission in 2005 is significantly less – roughly 1 billion dollars.

**1.29. IDENTIFY:** The cost would equal the number of dollar bills required; the surface area of the U.S. divided by the surface area of a single dollar bill.

**SET UP:** By drawing a rectangle on a map of the U.S., the approximate area is 2600 mi by 1300 mi or  $3,380,000 \text{ mi}^2$ . This estimate is within 10 percent of the actual area,  $3,794,083 \text{ mi}^2$ . The population is roughly  $3.0 \times 10^8$  while the area of a dollar bill, as measured with a ruler, is approximately  $6\frac{1}{8} \text{ in.}$  by  $2\frac{5}{8} \text{ in.}$

**EXECUTE:**  $A_{\text{U.S.}} = (3,380,000 \text{ mi}^2) [(5280 \text{ ft}) / (1 \text{ mi})]^2 [(12 \text{ in.}) / (1 \text{ ft})]^2 = 1.4 \times 10^{16} \text{ in.}^2$

$$A_{\text{bill}} = (6.125 \text{ in.})(2.625 \text{ in.}) = 16.1 \text{ in.}^2$$

$$\text{Total cost} = N_{\text{bills}} = A_{\text{U.S.}} / A_{\text{bill}} = (1.4 \times 10^{16} \text{ in.}^2) / (16.1 \text{ in.}^2 / \text{bill}) = 9 \times 10^{14} \text{ bills}$$

$$\text{Cost per person} = (9 \times 10^{14} \text{ dollars}) / (3.0 \times 10^8 \text{ persons}) = 3 \times 10^6 \text{ dollars/person}$$

**EVALUATE:** The actual cost would be somewhat larger, because the land isn't flat.

**1.30. IDENTIFY:** The displacements must be added as vectors and the magnitude of the sum depends on the relative orientation of the two displacements.

**SET UP:** The sum with the largest magnitude is when the two displacements are parallel and the sum with the smallest magnitude is when the two displacements are antiparallel.

**EXECUTE:** The orientations of the displacements that give the desired sum are shown in Figure 1.30.

**EVALUATE:** The orientations of the two displacements can be chosen such that the sum has any value between 0.6 m and 4.2 m.

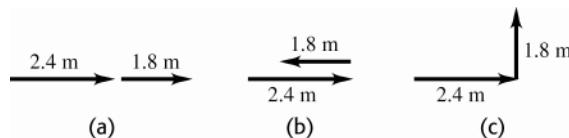


Figure 1.30

- 1.31. IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.
- SET UP:** Call the three displacements  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$ . The resultant displacement  $\vec{R}$  is given by  $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ .
- EXECUTE:** The vector addition diagram is given in Figure 1.31. Careful measurement gives that  $\vec{R}$  is 7.8 km,  $38^\circ$  north of east.
- EVALUATE:** The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements,  $2.6 \text{ km} + 4.0 \text{ km} + 3.1 \text{ km}$ .

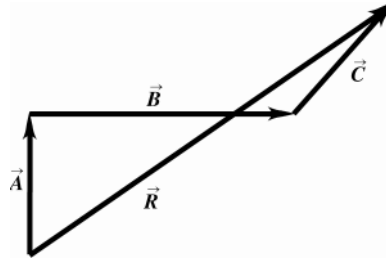


Figure 1.31

- 1.32. IDENTIFY:** Draw the vector addition diagram, so scale.
- SET UP:** The two vectors  $\vec{A}$  and  $\vec{B}$  are specified in the figure that accompanies the problem.
- EXECUTE:** (a) The diagram for  $\vec{C} = \vec{A} + \vec{B}$  is given in Figure 1.32a. Measuring the length and angle of  $\vec{C}$  gives  $C = 9.0 \text{ m}$  and an angle of  $\theta = 34^\circ$ .
- (b) The diagram for  $\vec{D} = \vec{A} - \vec{B}$  is given in Figure 1.32b. Measuring the length and angle of  $\vec{D}$  gives  $D = 22 \text{ m}$  and an angle of  $\theta = 250^\circ$ .
- (c)  $-\vec{A} - \vec{B} = -(\vec{A} + \vec{B})$ , so  $-\vec{A} - \vec{B}$  has a magnitude of  $9.0 \text{ m}$  (the same as  $\vec{A} + \vec{B}$ ) and an angle with the  $+x$  axis of  $214^\circ$  (opposite to the direction of  $\vec{A} + \vec{B}$ ).
- (d)  $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$ , so  $\vec{B} - \vec{A}$  has a magnitude of  $22 \text{ m}$  and an angle with the  $+x$  axis of  $70^\circ$  (opposite to the direction of  $\vec{A} - \vec{B}$ ).
- EVALUATE:** The vector  $-\vec{A}$  is equal in magnitude and opposite in direction to the vector  $\vec{A}$ .

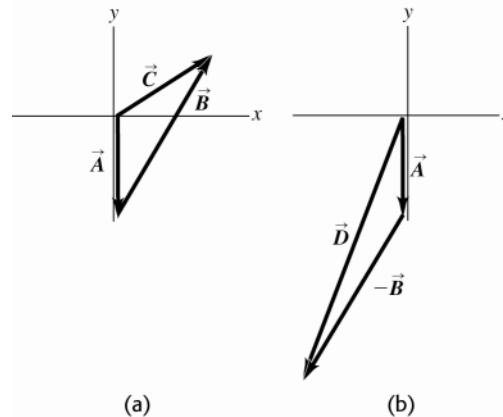


Figure 1.32

- 1.33. IDENTIFY:** Since she returns to the starting point, the vectors sum of the four displacements must be zero.
- SET UP:** Call the three given displacements  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$ , and call the fourth displacement  $\vec{D}$ .  $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$ .
- EXECUTE:** The vector addition diagram is sketched in Figure 1.33. Careful measurement gives that  $\vec{D}$  is  $144 \text{ m}$ ,  $41^\circ$  south of west.

**EVALUATE:**  $\vec{D}$  is equal in magnitude and opposite in direction to the sum  $\vec{A} + \vec{B} + \vec{C}$ .

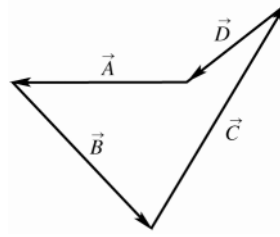


Figure 1.33

**1.34. IDENTIFY and SET UP:** Use a ruler and protractor to draw the vectors described. Then draw the corresponding horizontal and vertical components.

**EXECUTE:** (a) Figure 1.34 gives components 4.7 m, 8.1 m.

(b) Figure 1.34 gives components  $-15.6$  km, 15.6 km.

(c) Figure 1.34 gives components 3.82 cm,  $-5.07$  cm.

**EVALUATE:** The signs of the components depend on the quadrant in which the vector lies.

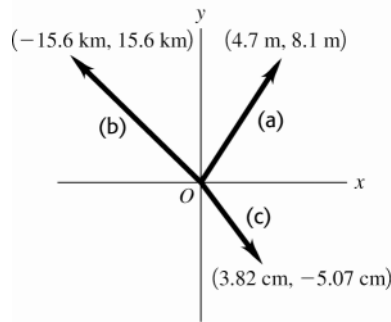


Figure 1.34

**1.35. IDENTIFY:** For each vector  $\vec{V}$ , use that  $V_x = V \cos \theta$  and  $V_y = V \sin \theta$ , when  $\theta$  is the angle  $\vec{V}$  makes with the  $+x$  axis, measured counterclockwise from the axis.

**SET UP:** For  $\vec{A}$ ,  $\theta = 270.0^\circ$ . For  $\vec{B}$ ,  $\theta = 60.0^\circ$ . For  $\vec{C}$ ,  $\theta = 205.0^\circ$ . For  $\vec{D}$ ,  $\theta = 143.0^\circ$ .

**EXECUTE:**  $A_x = 0$ ,  $A_y = -8.00$  m.  $B_x = 7.50$  m,  $B_y = 13.0$  m.  $C_x = -10.9$  m,  $C_y = -5.07$  m.  $D_x = -7.99$  m,  $D_y = 6.02$  m.

**EVALUATE:** The signs of the components correspond to the quadrant in which the vector lies.

**1.36. IDENTIFY:**  $\tan \theta = \frac{A_y}{A_x}$ , for  $\theta$  measured counterclockwise from the  $+x$ -axis.

**SET UP:** A sketch of  $A_x$ ,  $A_y$  and  $\vec{A}$  tells us the quadrant in which  $\vec{A}$  lies.

**EXECUTE:**

$$(a) \tan \theta = \frac{A_y}{A_x} = \frac{-1.00 \text{ m}}{2.00 \text{ m}} = -0.500. \theta = \tan^{-1}(-0.500) = 360^\circ - 26.6^\circ = 333^\circ.$$

$$(b) \tan \theta = \frac{A_y}{A_x} = \frac{1.00 \text{ m}}{2.00 \text{ m}} = 0.500. \theta = \tan^{-1}(0.500) = 26.6^\circ.$$

$$(c) \tan \theta = \frac{A_y}{A_x} = \frac{1.00 \text{ m}}{-2.00 \text{ m}} = -0.500. \theta = \tan^{-1}(-0.500) = 180^\circ - 26.6^\circ = 153^\circ.$$

$$(d) \tan \theta = \frac{A_y}{A_x} = \frac{-1.00 \text{ m}}{-2.00 \text{ m}} = 0.500. \theta = \tan^{-1}(0.500) = 180^\circ + 26.6^\circ = 207^\circ$$

**EVALUATE:** The angles  $26.6^\circ$  and  $207^\circ$  have the same tangent. Our sketch tells us which is the correct value of  $\theta$ .

**1.37. IDENTIFY:** Find the vector sum of the two forces.

**SET UP:** Use components to add the two forces. Take the  $+x$ -direction to be forward and the  $+y$ -direction to be upward.

**EXECUTE:** The second force has components  $F_{2x} = F_2 \cos 32.4^\circ = 433 \text{ N}$  and  $F_{2y} = F_2 \sin 32.4^\circ = 275 \text{ N}$ . The first force has components  $F_{1x} = 725 \text{ N}$  and  $F_{1y} = 0$ .

$$F_x = F_{1x} + F_{2x} = 1158 \text{ N} \quad \text{and} \quad F_y = F_{1y} + F_{2y} = 275 \text{ N}$$

The resultant force is 1190 N in the direction  $13.4^\circ$  above the forward direction.

**EVALUATE:** Since the two forces are not in the same direction the magnitude of their vector sum is less than the sum of their magnitudes.

- 1.38. IDENTIFY:** Find the vector sum of the three given displacements.

**SET UP:** Use coordinates for which  $+x$  is east and  $+y$  is north. The driver's vector displacements are:

$$\vec{A} = 2.6 \text{ km}, 0^\circ \text{ of north}; \quad \vec{B} = 4.0 \text{ km}, 0^\circ \text{ of east}; \quad \vec{C} = 3.1 \text{ km}, 45^\circ \text{ north of east}.$$

**EXECUTE:**  $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45^\circ) = 6.2 \text{ km}$ ;  $R_y = A_y + B_y + C_y =$

$$2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45^\circ) = 4.8 \text{ km}; \quad R = \sqrt{R_x^2 + R_y^2} = 7.8 \text{ km}; \quad \theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38^\circ;$$

$\vec{R} = 7.8 \text{ km}, 38^\circ$  north of east. This result is confirmed by the sketch in Figure 1.38.

**EVALUATE:** Both  $R_x$  and  $R_y$  are positive and  $\vec{R}$  is in the first quadrant.

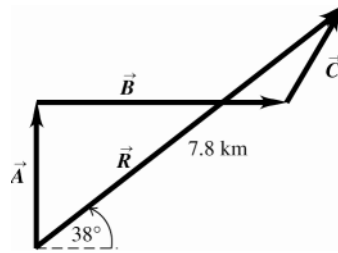


Figure 1.38

- 1.39. IDENTIFY:** If  $\vec{C} = \vec{A} + \vec{B}$ , then  $C_x = A_x + B_x$  and  $C_y = A_y + B_y$ . Use  $C_x$  and  $C_y$  to find the magnitude and direction of  $\vec{C}$ .

**SET UP:** From Figure 1.34 in the textbook,  $A_x = 0$ ,  $A_y = -8.00 \text{ m}$  and  $B_x = +B \sin 30.0^\circ = 7.50 \text{ m}$ ,

$$B_y = +B \cos 30.0^\circ = 13.0 \text{ m}.$$

**EXECUTE:** (a)  $\vec{C} = \vec{A} + \vec{B}$  so  $C_x = A_x + B_x = 7.50 \text{ m}$  and  $C_y = A_y + B_y = +5.00 \text{ m}$ .  $C = 9.01 \text{ m}$ .

$$\tan \theta = \frac{C_y}{C_x} = \frac{5.00 \text{ m}}{7.50 \text{ m}} \quad \text{and} \quad \theta = 33.7^\circ.$$

(b)  $\vec{B} + \vec{A} = \vec{A} + \vec{B}$ , so  $\vec{B} + \vec{A}$  has magnitude 9.01 m and direction specified by  $33.7^\circ$ .

(c)  $\vec{D} = \vec{A} - \vec{B}$  so  $D_x = A_x - B_x = -7.50 \text{ m}$  and  $D_y = A_y - B_y = -21.0 \text{ m}$ .  $D = 22.3 \text{ m}$ .  $\tan \phi = \frac{D_y}{D_x} = \frac{-21.0 \text{ m}}{-7.50 \text{ m}}$  and

$\phi = 70.3^\circ$ .  $\vec{D}$  is in the 3<sup>rd</sup> quadrant and the angle  $\theta$  counterclockwise from the  $+x$  axis is  $180^\circ + 70.3^\circ = 250.3^\circ$ .

(d)  $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$ , so  $\vec{B} - \vec{A}$  has magnitude 22.3 m and direction specified by  $\theta = 70.3^\circ$ .

**EVALUATE:** These results agree with those calculated from a scale drawing in Problem 1.32.

- 1.40. IDENTIFY:** Use Equations (1.7) and (1.8) to calculate the magnitude and direction of each of the given vectors.

**SET UP:** A sketch of  $A_x$ ,  $A_y$  and  $\vec{A}$  tells us the quadrant in which  $\vec{A}$  lies.

**EXECUTE:** (a)  $\sqrt{(-8.60 \text{ cm})^2 + (5.20 \text{ cm})^2} = 10.0 \text{ cm}$ ,  $\arctan\left(\frac{5.20}{-8.60}\right) = 148.8^\circ$  (which is  $180^\circ - 31.2^\circ$ ).

(b)  $\sqrt{(-9.7 \text{ m})^2 + (-2.45 \text{ m})^2} = 10.0 \text{ m}$ ,  $\arctan\left(\frac{-2.45}{-9.7}\right) = 14^\circ + 180^\circ = 194^\circ$ .

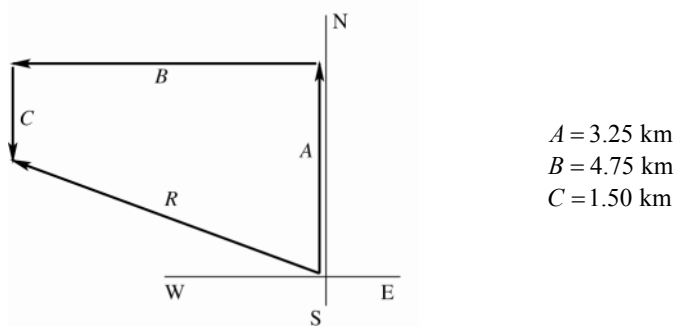
(c)  $\sqrt{(7.75 \text{ km})^2 + (-2.70 \text{ km})^2} = 8.21 \text{ km}$ ,  $\arctan\left(\frac{-2.7}{7.75}\right) = 340.8^\circ$  (which is  $360^\circ - 19.2^\circ$ ).

**EVALUATE:** In each case the angle is measured counterclockwise from the  $+x$  axis. Our results for  $\theta$  agree with our sketches.



**1.41. IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

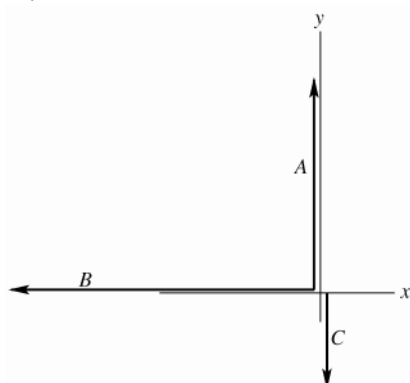
**SET UP:**



**Figure 1.41a**

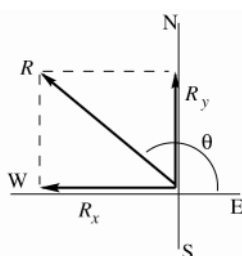
Select a coordinate system where  $+x$  is east and  $+y$  is north. Let  $\vec{A}$ ,  $\vec{B}$  and  $\vec{C}$  be the three displacements of the professor. Then the resultant displacement  $\vec{R}$  is given by  $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ . By the method of components,  $R_x = A_x + B_x + C_x$  and  $R_y = A_y + B_y + C_y$ . Find the  $x$  and  $y$  components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its  $x$  and  $y$  components that we have calculated. As always it is essential to draw a sketch.

**EXECUTE:**



**Figure 1.41b**

$$\begin{aligned} A_x &= 0, & A_y &= +3.25 \text{ km} \\ B_x &= -4.75 \text{ km}, & B_y &= 0 \\ C_x &= 0, & C_y &= -1.50 \text{ km} \\ R_x &= A_x + B_x + C_x \\ R_x &= 0 - 4.75 \text{ km} + 0 = -4.75 \text{ km} \\ R_y &= A_y + B_y + C_y \\ R_y &= 3.25 \text{ km} + 0 - 1.50 \text{ km} = 1.75 \text{ km} \end{aligned}$$



**Figure 1.41c**

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} = \sqrt{(-4.75 \text{ km})^2 + (1.75 \text{ km})^2} \\ R &= 5.06 \text{ km} \\ \tan \theta &= \frac{R_y}{R_x} = \frac{1.75 \text{ km}}{-4.75 \text{ km}} = -0.3684 \\ \theta &= 159.8^\circ \end{aligned}$$

The angle  $\theta$  measured counterclockwise from the  $+x$ -axis. In terms of compass directions, the resultant displacement is  $20.2^\circ$  N of W.

**EVALUATE:**  $R_x < 0$  and  $R_y > 0$ , so  $\vec{R}$  is in 2nd quadrant. This agrees with the vector addition diagram.

**1.42. IDENTIFY:** Add the vectors using components.  $\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$ .

**SET UP:** If  $\vec{C} = \vec{A} + \vec{B}$  then  $C_x = A_x + B_x$  and  $C_y = A_y + B_y$ . If  $\vec{D} = \vec{B} - \vec{A}$  then  $D_x = B_x - A_x$  and  $D_y = B_y - A_y$ .

**EXECUTE:** (a) The  $x$ - and  $y$ -components of the sum are  $1.30 \text{ cm} + 4.10 \text{ cm} = 5.40 \text{ cm}$ ,  $2.25 \text{ cm} + (-3.75 \text{ cm}) = -1.50 \text{ cm}$ .

(b) Using Equations (1.7) and (1.8),  $\sqrt{(5.40 \text{ cm})^2 + (-1.50 \text{ cm})^2} = 5.60 \text{ cm}$ ,  $\arctan\left(\frac{-1.50}{+5.40}\right) = 344.5^\circ$  ccw.

(c) Similarly,  $4.10 \text{ cm} - (1.30 \text{ cm}) = 2.80 \text{ cm}$ ,  $-3.75 \text{ cm} - (2.25 \text{ cm}) = -6.00 \text{ cm}$ .

(d)  $\sqrt{(2.80 \text{ cm})^2 + (-6.00 \text{ cm})^2} = 6.62 \text{ cm}$ ,  $\arctan\left(\frac{-6.00}{2.80}\right) = 295^\circ$  (which is  $360^\circ - 65^\circ$ ).

**EVALUATE:** We can draw the vector addition diagram in each case and verify that our results are qualitatively correct.

**1.43. IDENTIFY:** Vector addition problem.  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ .

**SET UP:** Find the  $x$ - and  $y$ -components of  $\vec{A}$  and  $\vec{B}$ . Then the  $x$ - and  $y$ -components of the vector sum are calculated from the  $x$ - and  $y$ -components of  $\vec{A}$  and  $\vec{B}$ .

**EXECUTE:**

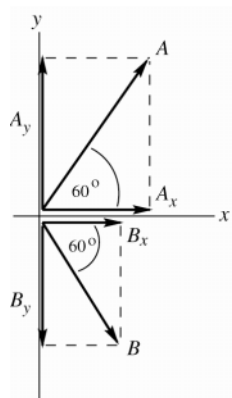


Figure 1.43a

$$A_x = A \cos(60.0^\circ)$$

$$A_x = (2.80 \text{ cm}) \cos(60.0^\circ) = +1.40 \text{ cm}$$

$$A_y = A \sin(60.0^\circ)$$

$$A_y = (2.80 \text{ cm}) \sin(60.0^\circ) = +2.425 \text{ cm}$$

$$B_x = B \cos(-60.0^\circ)$$

$$B_x = (1.90 \text{ cm}) \cos(-60.0^\circ) = +0.95 \text{ cm}$$

$$B_y = B \sin(-60.0^\circ)$$

$$B_y = (1.90 \text{ cm}) \sin(-60.0^\circ) = -1.645 \text{ cm}$$

Note that the signs of the components correspond to the directions of the component vectors.

(a) Now let  $\vec{R} = \vec{A} + \vec{B}$ .

$$R_x = A_x + B_x = +1.40 \text{ cm} + 0.95 \text{ cm} = +2.35 \text{ cm}.$$

$$R_y = A_y + B_y = +2.425 \text{ cm} - 1.645 \text{ cm} = +0.78 \text{ cm}.$$

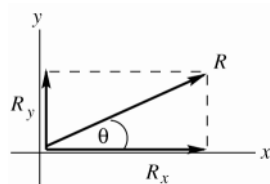


Figure 1.43b

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35 \text{ cm})^2 + (0.78 \text{ cm})^2}$$

$$R = 2.48 \text{ cm}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{+0.78 \text{ cm}}{+2.35 \text{ cm}} = +0.3319$$

$$\theta = 18.4^\circ$$

**EVALUATE:** The vector addition diagram for  $\vec{R} = \vec{A} + \vec{B}$  is

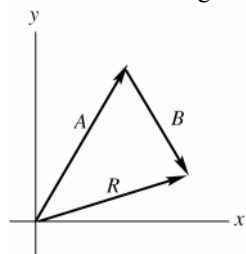


Figure 1.43c

$\vec{R}$  is in the 1st quadrant, with  $|R_y| < |R_x|$ , in agreement with our calculation.

**(b) EXECUTE:** Now let  $\vec{R} = \vec{A} - \vec{B}$ .

$$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$$

$$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$$

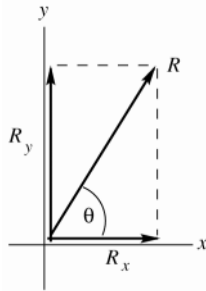


Figure 1.43d

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

$$R = 4.09 \text{ cm}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$$\theta = 83.7^\circ$$

**EVALUATE:** The vector addition diagram for  $\vec{R} = \vec{A} + (-\vec{B})$  is

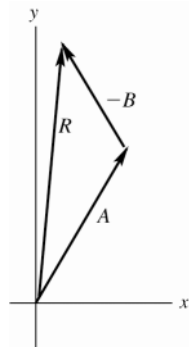


Figure 1.43e

$\vec{R}$  is in the 1st quadrant, with  $|R_x| < |R_y|$ , in agreement with our calculation.

**(c) EXECUTE:**

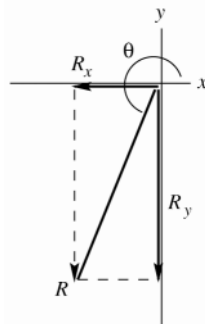


Figure 1.43f

$$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$$

$\vec{B} - \vec{A}$  and  $\vec{A} - \vec{B}$  are equal in magnitude and opposite in direction.

$$R = 4.09 \text{ cm} \text{ and}$$

$$\theta = 83.7^\circ + 180^\circ = 264^\circ$$

**EVALUATE:** The vector addition diagram for  $\vec{R} = \vec{B} + (-\vec{A})$  is

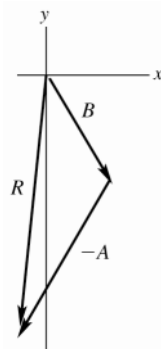


Figure 1.43g

$\vec{R}$  is in the 3rd quadrant, with  $|R_x| < |R_y|$ , in agreement with our calculation.

- 1.44. IDENTIFY:** The velocity of the boat relative to the earth,  $\vec{v}_{B/E}$ , the velocity of the water relative to the earth,  $\vec{v}_{W/E}$ , and the velocity of the boat relative to the water,  $\vec{v}_{B/W}$ , are related by  $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$ .

**SET UP:**  $\vec{v}_{W/E} = 5.0$  km/h, north and  $\vec{v}_{B/W} = 7.0$  km/h, west. The vector addition diagram is sketched in Figure 1.44.

**EXECUTE:**  $v_{B/E}^2 = v_{W/E}^2 + v_{B/W}^2$  and  $v_{B/E} = \sqrt{(5.0 \text{ km/h})^2 + (7.0 \text{ km/h})^2} = 8.6$  km/h.  $\tan \phi = \frac{v_{W/E}}{v_{B/W}} = \frac{5.0 \text{ km/h}}{7.0 \text{ km/h}}$  and

$\phi = 36^\circ$ , north of west.

**EVALUATE:** Since the two vectors we are adding are perpendicular we can use the Pythagorean theorem directly to find the magnitude of their vector sum.

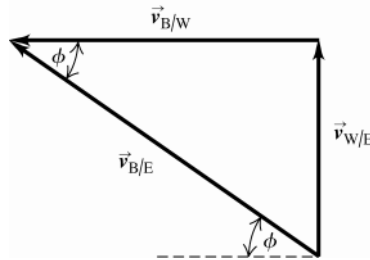


Figure 1.44

- 1.45. IDENTIFY:** Let  $A = 625$  N and  $B = 875$  N. We are asked to find the vector  $\vec{C}$  such that  $\vec{A} + \vec{B} + \vec{C} = 0$ .

**SET UP:**  $A_x = 0$ ,  $A_y = -625$  N.  $B_x = (875 \text{ N}) \cos 30^\circ = 758$  N,  $B_y = (875 \text{ N}) \sin 30^\circ = 438$  N.

**EXECUTE:**  $C_x = -(A_x + B_x) = -(0 + 758 \text{ N}) = -758$  N.  $C_y = -(A_y + B_y) = -(-625 \text{ N} + 438 \text{ N}) = +187$  N. Vector

$\vec{C}$  and its components are sketched in Figure 1.45.  $C = \sqrt{C_x^2 + C_y^2} = 781$  N.  $\tan \phi = \frac{|C_y|}{|C_x|} = \frac{187 \text{ N}}{758 \text{ N}}$  and  $\phi = 13.9^\circ$ .

$\vec{C}$  is at an angle of  $13.9^\circ$  above the  $-x$ -axis and therefore at an angle  $180^\circ - 13.9^\circ = 166.1^\circ$  counterclockwise from the  $+x$ -axis.

**EVALUATE:** A vector addition diagram for  $\vec{A} + \vec{B} + \vec{C}$  verifies that their sum is zero.

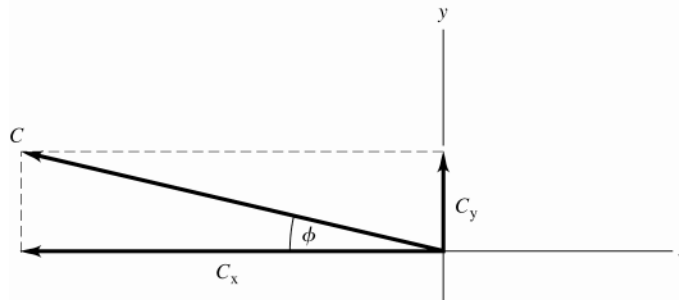


Figure 1.45

- 1.46. IDENTIFY:** We know the vector sum and want to find the magnitude of the vectors. Use the method of components.
- SET UP:** The two vectors  $\vec{A}$  and  $\vec{B}$  and their resultant  $\vec{C}$  are shown in Figure 1.46. Let  $+y$  be in the direction of the resultant.  $A = B$ .
- EXECUTE:**  $C_y = A_y + B_y$ .  $372 \text{ N} = 2A \cos 43.0^\circ$  and  $A = 254 \text{ N}$ .
- EVALUATE:** The sum of the magnitudes of the two forces exceeds the magnitude of the resultant force because only a component of each force is upward.

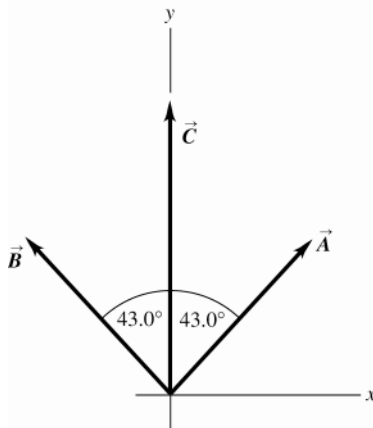


Figure 1.46

- 1.47. IDENTIFY:** Find the components of each vector and then use Eq.(1.14).
- SET UP:**  $A_x = 0$ ,  $A_y = -8.00 \text{ m}$ .  $B_x = 7.50 \text{ m}$ ,  $B_y = 13.0 \text{ m}$ .  $C_x = -10.9 \text{ m}$ ,  $C_y = -5.07 \text{ m}$ .  $D_x = -7.99 \text{ m}$ ,  $D_y = 6.02 \text{ m}$ .
- EXECUTE:**  $\vec{A} = (-8.00 \text{ m})\hat{j}$ ;  $\vec{B} = (7.50 \text{ m})\hat{i} + (13.0 \text{ m})\hat{j}$ ;  $\vec{C} = (-10.9 \text{ m})\hat{i} + (-5.07 \text{ m})\hat{j}$ ;  
 $\vec{D} = (-7.99 \text{ m})\hat{i} + (6.02 \text{ m})\hat{j}$ .
- EVALUATE:** All these vectors lie in the  $xy$ -plane and have no  $z$ -component.
- 1.48. IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is  $\vec{A} = A_x\hat{i} + A_y\hat{j}$
- SET UP:**  $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\hat{i} - 30\hat{j}$
- EXECUTE:** (a)  $A_x = 5.0$ ,  $A_y = -6.3$  (b)  $A_x = 11.2$ ,  $A_y = -9.91$  (c)  $A_x = -15.0$ ,  $A_y = 22.4$   
 (d)  $A_x = 20$ ,  $A_y = -30$
- EVALUATE:** The components are signed scalars.
- 1.49. IDENTIFY:** Use trig to find the components of each vector. Use Eq.(1.11) to find the components of the vector sum. Eq.(1.14) expresses a vector in terms of its components.
- SET UP:** Use the coordinates in the figure that accompanies the problem.
- EXECUTE:** (a)  $\vec{A} = (3.60 \text{ m})\cos 70.0^\circ\hat{i} + (3.60 \text{ m})\sin 70.0^\circ\hat{j} = (1.23 \text{ m})\hat{i} + (3.38 \text{ m})\hat{j}$   
 $\vec{B} = -(2.40 \text{ m})\cos 30.0^\circ\hat{i} - (2.40 \text{ m})\sin 30.0^\circ\hat{j} = (-2.08 \text{ m})\hat{i} + (-1.20 \text{ m})\hat{j}$   
 (b)  $\vec{C} = (3.00)\vec{A} - (4.00)\vec{B} = (3.00)(1.23 \text{ m})\hat{i} + (3.00)(3.38 \text{ m})\hat{j} - (4.00)(-2.08 \text{ m})\hat{i} - (4.00)(-1.20 \text{ m})\hat{j}$   
 $= (12.01 \text{ m})\hat{i} + (14.94)\hat{j}$   
 (c) From Equations (1.7) and (1.8),  
 $C = \sqrt{(12.01 \text{ m})^2 + (14.94 \text{ m})^2} = 19.17 \text{ m}$ ,  $\arctan\left(\frac{14.94 \text{ m}}{12.01 \text{ m}}\right) = 51.2^\circ$
- EVALUATE:**  $C_x$  and  $C_y$  are both positive, so  $\theta$  is in the first quadrant.
- 1.50. IDENTIFY:** Find  $A$  and  $B$ . Find the vector difference using components.
- SET UP:** Deduce the  $x$ - and  $y$ -components and use Eq.(1.8).
- EXECUTE:** (a)  $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$ ;  $A_x = +4.00$ ;  $A_y = +3.00$   
 $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (3.00)^2} = 5.00$

$$\vec{B} = 5.00\hat{i} - 2.00\hat{j}; B_x = +5.00; B_y = -2.00$$

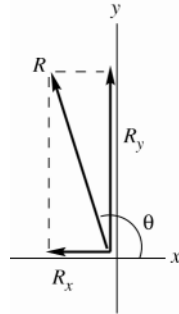
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$$

EVALUATE: Note that the magnitudes of  $\vec{A}$  and  $\vec{B}$  are each larger than either of their components.

EXECUTE: (b)  $\vec{A} - \vec{B} = 4.00\hat{i} + 3.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (3.00 + 2.00)\hat{j}$

$$\vec{A} - \vec{B} = -1.00\hat{i} + 5.00\hat{j}$$

(c) Let  $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 5.00\hat{j}$ . Then  $R_x = -1.00$ ,  $R_y = 5.00$ .



$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(-1.00)^2 + (5.00)^2} = 5.10.$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{5.00}{-1.00} = -5.00$$

$$\theta = -78.7^\circ + 180^\circ = 101.3^\circ.$$

Figure 1.50

EVALUATE:  $R_x < 0$  and  $R_y > 0$ , so  $\vec{R}$  is in the 2nd quadrant.

1.51. IDENTIFY: A unit vector has magnitude equal to 1.

SET UP: The magnitude of a vector is given in terms of its components by Eq.(1.12).

EXECUTE: (a)  $|\hat{i} + \hat{j} + \hat{k}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 1$  so it is not a unit vector.

(b)  $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ . If any component is greater than +1 or less than -1,  $|\vec{A}| > 1$ , so it cannot be a unit vector.  $\vec{A}$  can have negative components since the minus sign goes away when the component is squared.

(c)  $|\vec{A}| = 1$  gives  $\sqrt{a^2(3.0)^2 + a^2(4.0)^2} = 1$  and  $\sqrt{a^2}\sqrt{25} = 1$ .  $a = \pm \frac{1}{5.0} = \pm 0.20$ .

EVALUATE: The magnitude of a vector is greater than the magnitude of any of its components.

1.52. IDENTIFY: If vectors  $\vec{A}$  and  $\vec{B}$  commute for addition,  $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ . If they commute for the scalar product,  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ .

SET UP: Express the sum and scalar product in terms of the components of  $\vec{A}$  and  $\vec{B}$ .

EXECUTE: (a) Let  $\vec{A} = A_x\hat{i} + A_y\hat{j}$  and  $\vec{B} = B_x\hat{i} + B_y\hat{j}$ .  $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$ .

$$\vec{B} + \vec{A} = (B_x + A_x)\hat{i} + (B_y + A_y)\hat{j}. \text{ Scalar addition is commutative, so } \vec{A} + \vec{B} = \vec{B} + \vec{A}.$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y \text{ and } \vec{B} \cdot \vec{A} = B_x A_x + B_y A_y. \text{ Scalar multiplication is commutative, so } \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}.$$

(b)  $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$ .

$\vec{B} \times \vec{A} = (B_y A_z - B_z A_y)\hat{i} + (B_z A_x - B_x A_z)\hat{j} + (B_x A_y - B_y A_x)\hat{k}$ . Comparison of each component in each vector product shows that one is the negative of the other.

EVALUATE: The result in part (b) means that  $\vec{A} \times \vec{B}$  and  $\vec{B} \times \vec{A}$  have the same magnitude and opposite direction.

1.53. IDENTIFY:  $\vec{A} \cdot \vec{B} = AB \cos \phi$

SET UP: For  $\vec{A}$  and  $\vec{B}$ ,  $\phi = 150.0^\circ$ . For  $\vec{B}$  and  $\vec{C}$ ,  $\phi = 145.0^\circ$ . For  $\vec{A}$  and  $\vec{C}$ ,  $\phi = 65.0^\circ$ .

EXECUTE: (a)  $\vec{A} \cdot \vec{B} = (8.00 \text{ m})(15.0 \text{ m})\cos 150.0^\circ = -104 \text{ m}^2$

(b)  $\vec{B} \cdot \vec{C} = (15.0 \text{ m})(12.0 \text{ m})\cos 145.0^\circ = -148 \text{ m}^2$

(c)  $\vec{A} \cdot \vec{C} = (8.00 \text{ m})(12.0 \text{ m})\cos 65.0^\circ = 40.6 \text{ m}^2$

EVALUATE: When  $\phi < 90^\circ$  the scalar product is positive and when  $\phi > 90^\circ$  the scalar product is negative.

1.54. IDENTIFY: Target variables are  $\vec{A} \cdot \vec{B}$  and the angle  $\phi$  between the two vectors.

SET UP: We are given  $\vec{A}$  and  $\vec{B}$  in unit vector form and can take the scalar product using Eq.(1.19). The angle  $\phi$  can then be found from Eq.(1.18).