EXECUTE:

(a) $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 5.00$, $B = 5.39$

$\vec{A}\cdot\vec{B} = (4.00\hat{i} + 3.00\hat{j})\cdot(5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (3.00)(-2.00) = 20.0 - 6.0 = +14.0.$

(b) $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{14.0}{(5.00)(5.39)} = 0.519;\ \ \phi = 58.7°.$

EVALUATE: The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle ϕ is less than $90°$.

1.55. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as $\phi = \arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\dfrac{A_x B_x + A_y B_y}{AB}\right)$.

SET UP: Eq.(1.14) shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\dfrac{-22}{\sqrt{40}\,\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\phi = \arccos\left(\dfrac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.56. **IDENTIFY:** $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $|\vec{A}\times\vec{B}| = AB\sin\phi$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

SET UP: Figure 1.56 shows $\vec{A}$ and $\vec{B}$. The components $A_{\parallel}$ of $\vec{A}$ along $\vec{B}$ and $A_{\perp}$ of $\vec{A}$ perpendicular to $\vec{B}$ are shown in Figure 1.56a. The components of $B_{\parallel}$ of $\vec{B}$ along $\vec{A}$ and $B_{\perp}$ of $\vec{B}$ perpendicular to $\vec{A}$ are shown in Figure 1.56b.

EXECUTE: **(a)** From Figures 1.56a and b, $A_{\parallel} = A\cos\phi$ and $B_{\parallel} = B\cos\phi$. $\vec{A}\cdot\vec{B} = AB\cos\phi = BA_{\parallel} = AB_{\parallel}$.

(b) $A_{\perp} = A\sin\phi$ and $B_{\perp} = B\sin\phi$. $|\vec{A}\times\vec{B}| = AB\sin\phi = BA_{\perp} = AB_{\perp}$.

EVALUATE: When $\vec{A}$ and $\vec{B}$ are perpendicular, $\vec{A}$ has no component along $\vec{B}$ and $\vec{B}$ has no component along $\vec{A}$ and $\vec{A}\cdot\vec{B} = 0$. When $\vec{A}$ and $\vec{B}$ are parallel, $\vec{A}$ has no component perpendicular to $\vec{B}$ and $\vec{B}$ has no component perpendicular to $\vec{A}$ and $|\vec{A}\times\vec{B}| = 0$.

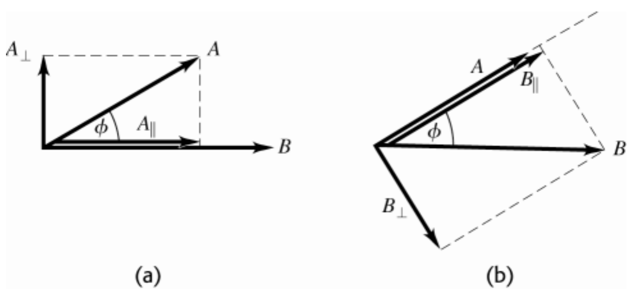


Figure 1.56

1.57. **IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

SET UP: $\phi = 180° - 53° = 127°$

EXECUTE: $|\vec{A}\times\vec{D}| = (8.00\text{ m})(10.0\text{ m})\sin127° = 63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

EVALUATE: The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_{\perp} = D\sin 53.0° = 7.00\text{ m}$. $|\vec{A}\times\vec{D}| = AD_{\perp} = 63.9\text{ m}^2$, which agrees with our previous result.

1.58. **IDENTIFY:** Target variable is the vector $\vec{A}\times\vec{B}$, expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using Eq.(1.24).

EXECUTE: $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$

$\vec{A}\times\vec{B}=(4.00\hat{i}+3.00\hat{j})\times(5.00\hat{i}-2.00\hat{j})=20.0\hat{i}\times\hat{i}-8.00\hat{i}\times\hat{j}+15.0\hat{j}\times\hat{i}-6.00\hat{j}\times\hat{j}$

But $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=0$ and $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{i}=-\hat{k}$, so $\vec{A}\times\vec{B}=-8.00\hat{k}+15.0(-\hat{k})=-23.0\hat{k}$.

The magnitude of $\vec{A}\times\vec{B}$ is 23.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the *xy*-plane is in the plane of the paper and the *z*-axis is directed out toward you.

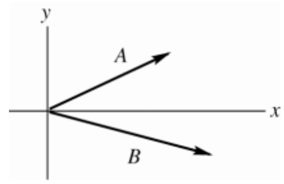


Figure 1.58

By the right-hand rule $\vec{A}\times\vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

1.59. **IDENTIFY:** The right-hand rule gives the direction and Eq.(1.22) gives the magnitude.

SET UP: $\phi=120.0°$.

EXECUTE: **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is $AB\sin\phi=(2.80\text{ cm})(1.90\text{ cm})\sin120^\circ=4.61\text{ cm}^2$.

(b) Rather than repeat the calculations, Eq. (1.23) may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

EVALUATE: For part (a) we could use Eq. (1.27) and note that the only non-vanishing component is

$$C_z=A_xB_y-A_yB_x=(2.80\text{ cm})\cos60.0°(-1.90\text{ cm})\sin60°$$
$$-(2.80\text{ cm})\sin60.0°(1.90\text{ cm})\cos60.0°=-4.61\text{ cm}^2.$$

This gives the same result.

1.60. **IDENTIFY:** Area is length times width. Do unit conversions.

SET UP: $1\text{ mi}=5280\text{ ft}$. $1\text{ ft}^3=7.477\text{ gal}$.

EXECUTE: **(a)** The area of one acre is $\frac{1}{8}\text{ mi}\times\frac{1}{80}\text{ mi}=\frac{1}{640}\text{ mi}^2$, so there are 640 acres to a square mile.

(b) $(1\text{ acre})\times\left(\dfrac{1\text{ mi}^2}{640\text{ acre}}\right)\times\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right)^2=43{,}560\text{ ft}^2$

(all of the above conversions are exact).

(c) $(1\text{ acre-foot})=(43{,}560\text{ ft}^3)\times\left(\dfrac{7.477\text{ gal}}{1\text{ ft}^3}\right)=3.26\times10^5\text{ gal}$, which is rounded to three significant figures.

EVALUATE: An acre is much larger than a square foot but less than a square mile. A volume of 1 acre-foot is much larger than a gallon.

1.61. **IDENTIFY:** The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

SET UP: The earth has mass $m_E=5.97\times10^{24}\text{ kg}$ and radius $r_E=6.38\times10^6\text{ m}$. The volume of a sphere is $V=\frac{4}{3}\pi r^3$. $\rho=1.76\text{ g/cm}^3=1760\text{ km/m}^3$.

EXECUTE: **(a)** The planet has mass $m=5.5m_E=3.28\times10^{25}\text{ kg}$. $V=\dfrac{m}{\rho}=\dfrac{3.28\times10^{25}\text{ kg}}{1760\text{ kg/m}^3}=1.86\times10^{22}\text{ m}^3$.

$$r=\left(\frac{3V}{4\pi}\right)^{1/3}=\left(\frac{3[1.86\times10^{22}\text{ m}^3]}{4\pi}\right)^{1/3}=1.64\times10^7\text{ m}=1.64\times10^4\text{ km}$$

(b) $r=2.57r_E$

EVALUATE: Volume V is proportional to mass and radius r is proportional to $V^{1/3}$, so r is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_E=1.8r_E$. The radius of the planet is greater than this, so its density must be less than that of the earth.

1.62. **IDENTIFY** and **SET UP:** Unit conversion.

EXECUTE: **(a)** $f = 1.420\times10^9$ cycles/s, so $\dfrac{1}{1.420\times10^9}$ s $= 7.04\times10^{-10}$ s for one cycle.

(b) $\dfrac{3600 \text{ s/h}}{7.04\times10^{-10} \text{ s/cycle}} = 5.11\times10^{12}$ cycles/h

(c) Calculate the number of seconds in 4600 million years $= 4.6\times10^9$ y and divide by the time for 1 cycle:

$$\frac{(4.6\times10^9 \text{ y})(3.156\times10^7 \text{ s/y})}{7.04\times10^{-10} \text{ s/cycle}} = 2.1\times10^{26} \text{ cycles}$$

(d) The clock is off by 1 s in $100{,}000 \text{ y} = 1\times10^5$ y, so in 4.60×10^9 y it is off by $(1 \text{ s})\left(\dfrac{4.60\times10^9}{1\times10^5}\right) = 4.6\times10^4$ s (about 13 h).

EVALUATE: In each case the units in the calculation combine algebraically to give the correct units for the answer.

1.63. **IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.

SET UP: Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: $18.015 \text{ u}\times1.661\times10^{-27} \text{ kg/u} = 2.992\times10^{-26}$ kg/molecule.

EXECUTE: $(70 \text{ kg})/(2.992\times10^{-26} \text{ kg/molecule}) = 2.34\times10^{27}$ molecules. Each H_2O molecule has 3 atoms, so there are about 6×10^{27} atoms.

EVALUATE: Assuming carbon to be the most common atom gives 3×10^{27} molecules, which is a result of the same order of magnitude.

1.64. **IDENTIFY:** Estimate the volume of each object. The mass m is the density times the volume.

SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. The volume of a cylinder of radius r and length l is $V = \pi r^2 l$. The density of water is 1000 kg/m^3.

EXECUTE: **(a)** Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2\times10^{-4}\text{m}^3$.
$m = (0.98)(1000 \text{ kg/m}^3)(5.2\times10^{-4}\text{m}^3) = 0.5 \text{ kg}$.

(b) Approximate as a sphere of radius $r = 0.25\mu\text{m}$ (probably an over estimate): $V = 6.5\times10^{-20}\text{m}^3$.
$m = (0.98)(1000 \text{ kg/m}^3)(6.5\times10^{-20} \text{ m}^3) = 6\times10^{-17} \text{ kg} = 6\times10^{-14} \text{ g}$.

(c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8\times10^{-7}\text{m}^3$.
$m = (0.98)(1000 \text{ kg/m}^3)(2.8\times10^{-7} \text{ m}^3) = 3\times10^{-4} \text{ kg} = 0.3 \text{ g}$.

EVALUATE: The mass is directly proportional to the volume.

1.65. **IDENTIFY:** Use the volume V and density ρ to calculate the mass M: $\rho = \dfrac{M}{V}$, so $V = \dfrac{M}{\rho}$.

SET UP: The volume of a cube with sides of length x is x^3. The volume of a sphere with radius R is $\frac{4}{3}\pi R^3$.

EXECUTE: **(a)** $x^3 = \dfrac{0.200 \text{ kg}}{7.86\times10^3 \text{ kg/m}^3} = 2.54\times10^{-5}\text{m}^3$. $x = 2.94\times10^{-2}\text{m} = 2.94 \text{ cm}$.

(b) $\dfrac{4}{3}\pi R^3 = 2.54\times10^{-5}\text{m}^3$. $R = 1.82\times10^{-2}\text{m} = 1.82 \text{ cm}$.

EVALUATE: $\frac{4}{3}\pi = 4.2$, so a sphere with radius R has a greater volume than a cube whose sides have length R.

1.66. **IDENTIFY:** Estimate the volume of sand in all the beaches on the earth. The diameter of a grain of sand determines its volume. From the volume of one grain and the total volume of sand we can calculate the number of grains.

SET UP: The volume of a sphere of diameter d is $V = \frac{1}{6}\pi d^3$. Consulting an atlas, we estimate that the continents have about 1.45×10^5 km of coastline. Add another 25% of this for rivers and lakes, giving 1.82×10^5 km of coastline. Assume that a beach extends 50 m beyond the water and that the sand is 2 m deep. 1 billion $= 1\times10^9$.

EXECUTE: **(a)** The volume of sand is $(1.82\times10^8 \text{ m})(50 \text{ m})(2 \text{ m}) = 2\times10^{10} \text{ m}^3$. The volume of a grain is $V = \frac{1}{6}\pi(0.2\times10^{-3} \text{ m})^3 = 4\times10^{-12} \text{ m}^3$. The number of grains is $\dfrac{2\times10^{10} \text{ m}^3}{4\times10^{-12} \text{ m}^3} = 5\times10^{21}$. The number of grains of sand is about 10^{22}.

(b) The number of stars is $(100\times10^9)(100\times10^9) = 10^{22}$. The two estimates result in comparable numbers for these two quantities.

EVALUATE: Both numbers are crude estimates but are probably accurate to a few powers of 10.

1.67. IDENTIFY: The number of particles is the total mass divided by the mass of one particle.

SET UP: $1\text{ mol} = 6.0\times 10^{23}$ atoms. The mass of the earth is 6.0×10^{24} kg. The mass of the sun is 2.0×10^{30} kg. The distance from the earth to the sun is 1.5×10^{11} m. The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$. Protons and neutrons each have a mass of 1.7×10^{-27} kg and the mass of an electron is much less.

EXECUTE: **(a)** $(6.0\times 10^{24}\text{ kg})\times\left(\dfrac{6.0\times 10^{23}\ \frac{\text{atoms}}{\text{mole}}}{14\times 10^{-3}\ \frac{\text{kg}}{\text{mole}}}\right) = 2.6\times 10^{50}$ atoms.

(b) The number of neutrons is the mass of the neutron star divided by the mass of a neutron:

$\dfrac{(2)(2.0\times 10^{30}\text{ kg})}{(1.7\times 10^{-27}\text{ kg/neutron})} = 2.4\times 10^{57}$ neutrons.

(c) The average mass of a particle is essentially $\frac{2}{3}$ the mass of either the proton or the neutron, 1.7×10^{-27} kg. The total number of particles is the total mass divided by this average, and the total mass is the volume times the average density. Denoting the density by ρ,

$$\frac{M}{m_{\text{ave}}} = \frac{\frac{4}{3}\pi R^3\rho}{\frac{2}{3}m_{\text{p}}} = \frac{(2\pi)(1.5\times 10^{11}\text{ m})^3\ (10^{18}\text{ kg/m}^3)}{1.7\times 10^{-27}\text{ kg}} = 1.2\times 10^{79}.$$

Note the conversion from g/cm^3 to kg/m^3.

EVALUATE: These numbers of particles are each very, very large but are still much less than a googol.

1.68. IDENTIFY: Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, so $\vec{D}=-\left(\vec{A}+\vec{B}+\vec{C}\right)$.

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}$.

EXECUTE: $A_x = +A\cos 30.0° = +86.6\text{ N}$, $A_y = +A\cos 30.0° = +50.00\text{ N}$.

$B_x = -B\sin 30.0° = -40.00\text{ N}$, $B_y = +B\cos 30.0° = +69.28\text{ N}$.

$C_x = +C\cos 53.0° = -24.07\text{ N}$, $C_y = -C\sin 53.0° = -31.90\text{ N}$.

Then $D_x = -22.53$ N, $D_y = -87.34$ N and $D=\sqrt{D_x^2+D_y^2} = 90.2$ N. $\tan\alpha = |D_y/D_x| = 87.34/22.53$. $\alpha = 75.54°$. $\phi = 180° + \alpha = 256°$, counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.68, since D_x and D_y are both negative, $\vec{D}$ must lie in the third quadrant.

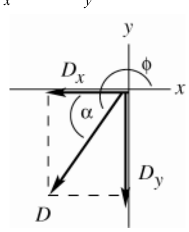


Figure 1.68

1.69. IDENTIFY: We know the magnitude and direction of the sum of the two vector pulls and the direction of one pull. We also know that one pull has twice the magnitude of the other. There are two unknowns, the magnitude of the smaller pull and its direction. $A_x + B_x = C_x$ and $A_y + B_y = C_y$ give two equations for these two unknowns.

SET UP: Let the smaller pull be $\vec{A}$ and the larger pull be $\vec{B}$. $B = 2A$. $\vec{C}=\vec{A}+\vec{B}$ has magnitude 350.0 N and is northward. Let $+x$ be east and $+y$ be north. $B_x = -B\sin 25.0°$ and $B_y = B\cos 25.0°$. $C_x = 0$, $C_y = 350.0$ N.

$\vec{A}$ must have an eastward component to cancel the westward component of $\vec{B}$. There are then two possibilities, as sketched in Figures 1.69 a and b. $\vec{A}$ can have a northward component or $\vec{A}$ can have a southward component.

EXECUTE: In either Figure 1.69 a or b, $A_x + B_x = C_x$ and $B = 2A$ gives $(2A)\sin 25.0° = A\sin\phi$ and $\phi = 57.7°$. In Figure 1.69a, $A_y + B_y = C_y$ gives $2A\cos 25.0° + A\cos 57.7° = 350.0$ N and $A = 149$ N. In Figure 1.69b, $2A\cos 25.0° - A\cos 57.7° = 350.0$ N and $A = 274$ N. One solution is for the smaller pull to be $57.7°$ east of north. In this case, the smaller pull is 149 N and the larger pull is 298 N. The other solution is for the smaller pull to be $57.7°$ east of south. In this case the smaller pull is 274 N and the larger pull is 548 N.

EVALUATE: For the first solution, with $\vec{A}$ east of north, each worker has to exert less force to produce the given resultant force and this is the sensible direction for the worker to pull.

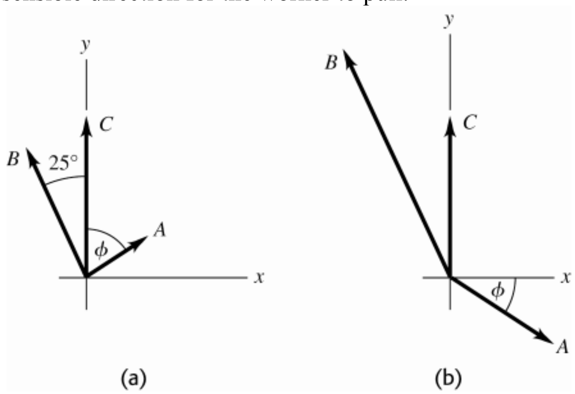


Figure 1.69

1.70. **IDENTIFY:** Find the vector sum of the two displacements.

SET UP: Call the two displacements $\vec{A}$ and $\vec{B}$, where $A = 170$ km and $B = 230$ km. $\vec{A} + \vec{B} = \vec{R}$. $\vec{A}$ and $\vec{B}$ are as shown in Figure 1.70.

EXECUTE: $R_x = A_x + B_x = (170 \text{ km}) \sin 68° + (230 \text{ km}) \cos 48° = 311.5 \text{ km}$.

$R_y = A_y + B_y = (170 \text{ km}) \cos 68° - (230 \text{ km}) \sin 48° = -107.2 \text{ km}$.

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(311.5 \text{ km})^2 + (-107.2 \text{ km})^2} = 330 \text{ km}$. $\tan\theta_R = \left|\frac{R_y}{R_x}\right| = \frac{107.2 \text{ km}}{311.5 \text{ km}} = 0.344$.

$\theta_R = 19°$ south of east**.**

EVALUATE: Our calculation using components agrees with $\vec{R}$ shown in the vector addition diagram, Figure 1.70.

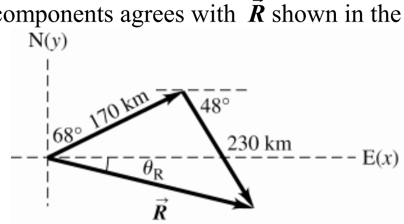


Figure 1.70

1.71. **IDENTIFY:** $\vec{A} + \vec{B} = \vec{C}$ (or $\vec{B} + \vec{A} = \vec{C}$). The target variable is vector $\vec{A}$.

SET UP: Use components and Eq.(1.10) to solve for the components of $\vec{A}$. Find the magnitude and direction of $\vec{A}$ from its components.

EXECUTE: **(a)**

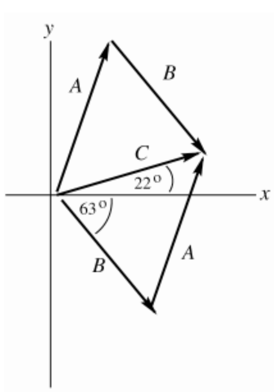


Figure 1.71a

$C_x = A_x + B_x$, so $A_x = C_x - B_x$

$C_y = A_y + B_y$, so $A_y = C_y - B_y$

$C_x = C\cos 22.0° = (6.40 \text{ cm})\cos 22.0°$

$C_x = +5.934$ cm

$C_y = C\sin 22.0° = (6.40 \text{ cm})\sin 22.0°$

$C_y = +2.397$ cm

$B_x = B\cos(360° - 63.0°) = (6.40 \text{ cm})\cos 297.0°$

$B_x = +2.906$ cm

$B_y = B\sin 297.0° = (6.40 \text{ cm})\sin 297.0°$

$B_y = -5.702$ cm

(b) $A_x = C_x - B_x = +5.934 \text{ cm} - 2.906 \text{ cm} = +3.03 \text{ cm}$

$A_y = C_y - B_y = +2.397 \text{ cm} - (-5.702) \text{ cm} = +8.10 \text{ cm}$

(c)

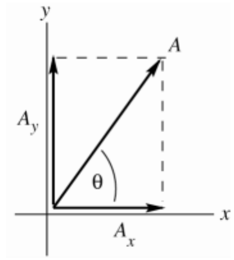


Figure 1.71b

$$A = \sqrt{A_x^2 + A_y^2}$$
$$A = \sqrt{(3.03\text{ cm})^2 + (8.10\text{ cm})^2} = 8.65\text{ cm}$$
$$\tan\theta = \frac{A_y}{A_x} = \frac{8.10\text{ cm}}{3.03\text{ cm}} = 2.67$$
$$\theta = 69.5°$$

EVALUATE: The $\vec{A}$ we calculated agrees qualitatively with vector $\vec{A}$ in the vector addition diagram in part (a).

1.72. **IDENTIFY:** Add the vectors using the method of components.

SET UP: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m.

EXECUTE: **(a)** $R_x = A_x + B_x + C_x = -3.4$ m. $R_y = A_y + B_y + C_y = -0.07$ m. $R = 3.4$ m. $\tan\theta = \dfrac{-0.07\text{ m}}{-3.4\text{ m}}$.

$\theta = 1.2°$ below the $-x$-axis.

(b) $S_x = C_x - A_x - B_x = -18.4$ m. $S_y = C_y - A_y - B_y = -10.1$ m. $S = 21.0$ m. $\tan\theta = \dfrac{S_y}{S_x} = \dfrac{-10.1\text{ m}}{-18.4\text{ m}}$. $\theta = 28.8°$ below the $-x$-axis.

EVALUATE: The magnitude and direction we calculated for $\vec{R}$ and $\vec{S}$ agree with our vector diagrams.

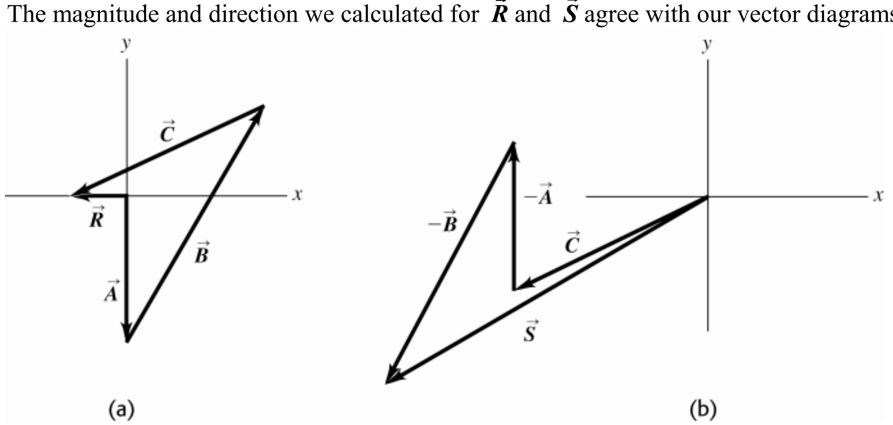


Figure 1.72

1.73. **IDENTIFY:** Vector addition. Target variable is the 4th displacement.

SET UP: Use a coordinate system where east is in the $+x$-direction and north is in the $+y$-direction.

Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements that are given and let $\vec{D}$ be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

EXECUTE:

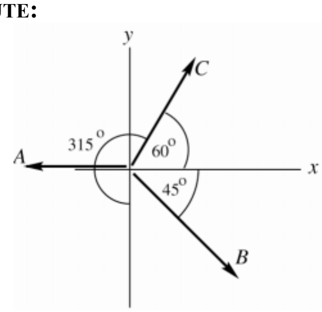


Figure 1.73a

$$A_x = -180\text{ m},\quad A_y = 0$$
$$B_x = B\cos 315° = (210\text{ m})\cos 315° = +148.5\text{ m}$$
$$B_y = B\sin 315° = (210\text{ m})\sin 315° = -148.5\text{ m}$$
$$C_x = C\cos 60° = (280\text{ m})\cos 60° = +140\text{ m}$$
$$C_y = C\sin 60° = (280\text{ m})\sin 60° = +242.5\text{ m}$$

$$D_x = -(A_x + B_x + C_x) = -(-180\text{ m} + 148.5\text{ m} + 140\text{ m}) = -108.5\text{ m}$$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5 \text{ m} + 242.5 \text{ m}) = -94.0 \text{ m}$

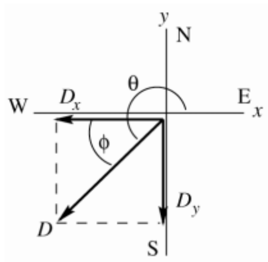


Figure 1.73b

$D = \sqrt{D_x^2 + D_y^2}$

$D = \sqrt{(-108.5 \text{ m})^2 + (-94.0 \text{ m})^2} = 144 \text{ m}$

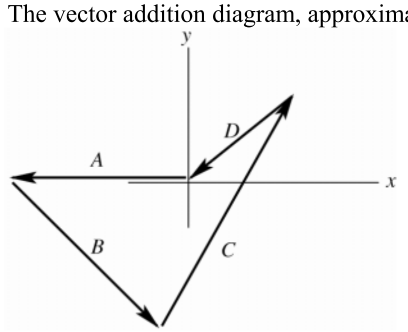
$\tan\theta = \dfrac{D_y}{D_x} = \dfrac{-94.0 \text{ m}}{-108.5 \text{ m}} = 0.8664$

$\theta = 180° + 40.9° = 220.9°$

($\vec{D}$ is in the third quadrant since both D_x and D_y are negative.)

The direction of $\vec{D}$ can also be specified in terms of $\phi = \theta - 180° = 40.9°$; $\vec{D}$ is $41°$ south of west.

EVALUATE: The vector addition diagram, approximately to scale, is



Vector $\vec{D}$ in this diagram agrees qualitatively with our calculation using components.

Figure 1.73c

1.74. IDENTIFY: Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80 \text{ km} - (2.00 \text{ km}) - (3.50 \text{ km})(\cos 45°) = 1.33 \text{ km}$; $C_y = R_y - A_y - B_y$ $= 0 \text{ km} - 0 \text{ km} - (-3.50 \text{ km})(\sin 45°) = 2.47 \text{ km}$; $C = \sqrt{(1.33 \text{ km})^2 + (2.47 \text{ km})^2} = 2.81 \text{ km}$; $\theta = \tan^{-1}\left[(2.47 \text{ km})/(1.33 \text{ km})\right] = 61.7°$ north of east. The vector addition diagram in Figure 1.74 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its *x* and *y* components are both positive.

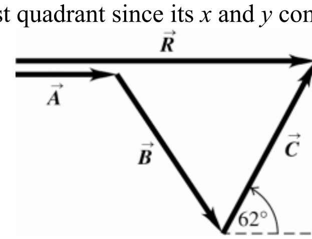


Figure 1.74

1.75. IDENTIFY: The sum of the vector forces on the beam sum to zero, so their *x* components and their *y* components sum to zero. Solve for the components of $\vec{F}$.

SET UP: The forces on the beam are sketched in Figure 1.75a. Choose coordinates as shown in the sketch. The 100-N pull makes an angle of $30.0° + 40.0° = 70.0°$ with the horizontal. $\vec{F}$ and the 100-N pull have been replaced by their *x* and *y* components.

EXECUTE: **(a)** The sum of the *x*-components is equal to zero gives $F_x + (100 \text{ N})\cos 70.0° = 0$ and $F_x = -34.2 \text{ N}$. The sum of the *y*-components is equal to zero gives $F_y + (100 \text{ N})\sin 70.0° - 124 \text{ N} = 0$ and $F_y = +30.0 \text{ N}$. $\vec{F}$ and its components are sketched in Figure 1.75b. $F = \sqrt{F_x^2 + F_y^2} = 45.5 \text{ N}$. $\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{30.0 \text{ N}}{34.2 \text{ N}}$ and $\phi = 41.3°$. $\vec{F}$ is directed at $41.3°$ above the $-x$-axis in Figure 1.75a.

(b) The vector addition diagram is given in Figure 1.75c. $\vec{F}$ determined from the diagram agrees with $\vec{F}$ calculated in part (a) using components.

EVALUATE: The vertical component of the 100 N pull is less than the 124 N weight so $\vec{F}$ must have an upward component if all three forces balance.

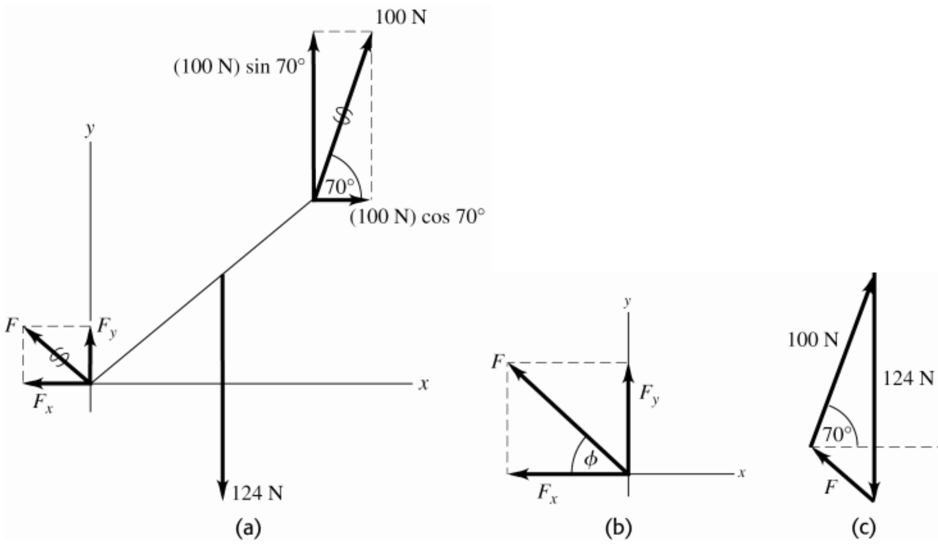


Figure 1.75

1.76. **IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where $\vec{A}$, $\vec{B}$ and $\vec{C}$ are in the three given displacements and $\vec{D}$ is the displacement for her return.

START UP: Let $+x$ be east and $+y$ be north.

EXECUTE: **(a)** $D_x = -[(147\text{ km})\sin 85° + (106\text{ km})\sin 167° + (166\text{ km})\sin 235°] = -34.3\text{ km}$.

$D_y = -[(147\text{ km})\cos 85° + (106\text{ km})\cos 167° + (166\text{ km})\cos 235°] = +185.7\text{ km}$.

$D = \sqrt{(-34.3\text{ km})^2 + (185.7\text{ km})^2} = 189\text{ km}$.

(b) The direction relative to north is $\phi = \arctan\left(\dfrac{34.3\text{ km}}{185.7\text{ km}}\right) = 10.5°$. Since $D_x < 0$ and $D_y > 0$, the direction of $\vec{D}$ is $10.5°$ west of north.

EVALUATE: The four displacements add to zero.

1.77. **IDENTIFY** and **SET UP:** The vector $\vec{A}$ that connects points (x_1, y_1) and (x_2, y_2) has components $A_x = x_2 - x_1$ and $A_y = y_2 - y_1$.

EXECUTE: **(a)** Angle of first line is $\theta = \tan^{-1}\left(\dfrac{200-20}{210-10}\right) = 42°$. Angle of second line is $42° + 30° = 72°$.

Therefore $X = 10 + 250\cos 72° = 87$, $Y = 20 + 250\sin 72° = 258$ for a final point of (87,258).

(b) The computer screen now looks something like Figure 1.77. The length of the bottom line is

$\sqrt{(210-87)^2 + (200-258)^2} = 136$ and its direction is $\tan^{-1}\left(\dfrac{258-200}{210-87}\right) = 25°$ below straight left.

EVALUATE: Figure 1.77 is a vector addition diagram. The vector first line plus the vector arrow gives the vector for the second line.

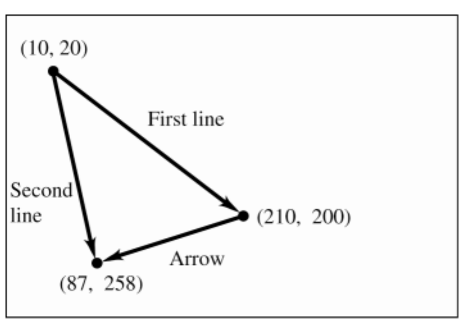


Figure 1.77

1.78. **IDENTIFY:** Let the three given displacements be $\vec{A}$, $\vec{B}$ and $\vec{C}$, where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement $\vec{C}$ that will return him to his hut is $-\vec{R}$.

SET UP: Let the east direction be the $+x$-direction and the north direction be the $+y$-direction.

EXECUTE: **(a)** The three displacements and their resultant are sketched in Figure 1.78.

(b) $R_x = (40)\cos 45° - (80)\cos\ 60° = -11.7$ and $R_y = (40)\sin 45° + (80)\sin 60° - 50 = 47.6$.

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, $\arctan\left(\frac{47.6}{11.7}\right) = 76°$, north of west.

We know that $\vec{R}$ is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction $76°$ south of east, which is $14°$ east of south.

EVALUATE: It is useful to show R_x, R_y and $\vec{R}$ on a sketch, so we can specify what angle we are computing.

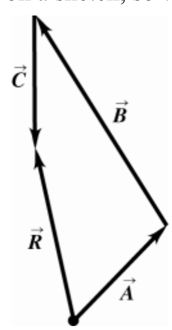


Figure 1.78

1.79. **IDENTIFY:** Vector addition. One vector and the sum are given; find the second vector (magnitude and direction).

SET UP: Let $+x$ be east and $+y$ be north. Let $\vec{A}$ be the displacement 285 km at $40.0°$ north of west and let $\vec{B}$ be the unknown displacement.

$\vec{A} + \vec{B} = \vec{R}$ where $\vec{R} = 115$ km, east

$\vec{B} = \vec{R} - \vec{A}$

$B_x = R_x - A_x,\ \ B_y = R_y - A_y$

EXECUTE: $A_x = -A\cos 40.0° = -218.3$ km, $A_y = +A\sin 40.0° = +183.2$ km

$R_x = 115$ km, $R_y = 0$

Then $B_x = 333.3$ km, $B_y = -183.2$ km. $B = \sqrt{B_x^2 + B_y^2} = 380$ km;

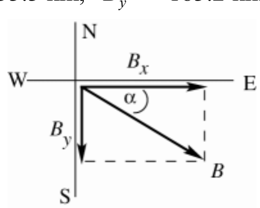


Figure 1.79

$\tan\alpha = |B_y / B_x| = (183.2\text{ km})/(333.3\text{ km})$

$\alpha = 28.8°$, south of east

EVALUATE: The southward component of $\vec{B}$ cancels the northward component of $\vec{A}$. The eastward component of $\vec{B}$ must be 115 km larger than the magnitude of the westward component of $\vec{A}$.

1.80. **IDENTIFY:** Find the components of the weight force, using the specified coordinate directions.

SET UP: For parts (a) and (b), take $+x$ direction along the hillside and the $+y$ direction in the downward direction and perpendicular to the hillside. For part (c), $\alpha = 35.0°$ and $w = 550$ N.

EXECUTE: **(a)** $w_x = w\sin\alpha$

(b) $w_y = w\cos\alpha$

(c) The maximum allowable weight is $w = w_x/(\sin\alpha) = (550\text{ N})/(\sin 35.0°) = 959$ N.

EVALUATE: The component parallel to the hill increases as α increases and the component perpendicular to the hill increases as α decreases.

1.81. **IDENTIFY:** Vector addition. One force and the vector sum are given; find the second force.
SET UP: Use components. Let $+y$ be upward.

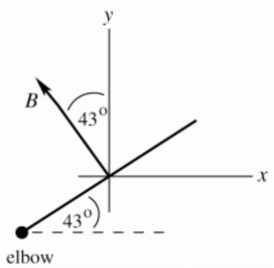


Figure 1.81a

$\vec{B}$ is the force the biceps exerts.

$\vec{E}$ is the force the elbow exerts. $\vec{E}+\vec{B}=\vec{R}$, where $R=132.5\text{ N}$ and is upward.

$E_x = R_x - B_x, \qquad E_y = R_y - B_y$

EXECUTE: $B_x = -B\sin 43° = -158.2\text{ N}$, $B_y = +B\cos 43° = +169.7\text{ N}$, $R_x = 0$, $R_y = +132.5\text{ N}$

Then $E_x = +158.2\text{ N}$, $E_y = -37.2\text{ N}$

$E = \sqrt{E_x^2 + E_y^2} = 160\text{ N};$

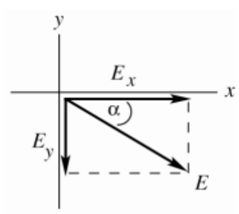


Figure 1.81b

$\tan\alpha = |E_y / E_x| = 37.2/158.2$

$\alpha = 13°$, below horizontal

EVALUATE: The x-component of $\vec{E}$ cancels the x-component of $\vec{B}$. The resultant upward force is less than the upward component of $\vec{B}$, so E_y must be downward.

1.82. **IDENTIFY:** Find the vector sum of the four displacements.
SET UP: Take the beginning of the journey as the origin, with north being the y-direction, east the x-direction, and the z-axis vertical. The first displacement is then $(-30\text{ m})\hat{\boldsymbol{k}}$, the second is $(-15\text{ m})\hat{\boldsymbol{j}}$, the third is $(200\text{ m})\hat{\boldsymbol{i}}$, and the fourth is $(100\text{ m})\hat{\boldsymbol{j}}$.
EXECUTE: **(a)** Adding the four displacements gives
$(-30\text{ m})\hat{\boldsymbol{k}} + (-15\text{ m})\hat{\boldsymbol{j}} + (200\text{ m})\hat{\boldsymbol{i}} + (100\text{ m})\hat{\boldsymbol{j}} = (200\text{ m})\hat{\boldsymbol{i}} + (85\text{ m})\hat{\boldsymbol{j}} - (30\text{ m})\hat{\boldsymbol{k}}.$
(b) The total distance traveled is the sum of the distances of the individual segments:
$30\text{ m} + 15\text{ m} + 200\text{ m} + 100\text{ m} = 345\text{ m}.$ The magnitude of the total displacement is:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(200\text{ m})^2 + (85\text{ m})^2 + \left(-30\text{ m}\right)^2} = 219\text{ m}.$$

EVALUATE: The magnitude of the displacement is much less than the distance traveled along the path.

1.83. **IDENTIFY:** The sum of the force displacements must be zero. Use components.
SET UP: Call the displacements $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$, where $\vec{D}$ is the final unknown displacement for the return from the treasure to the oak tree. Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are sketched in Figure 1.83a. $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$ says $A_x + B_x + C_x + D_x = 0$ and $A_y + B_y + C_y + D_y = 0$. $A = 825\text{ m}$, $B = 1250\text{ m}$, and $C = 1000\text{ m}$. Let $+x$ be eastward and $+y$ be north.
EXECUTE: **(a)** $A_x + B_x + C_x + D_x = 0$ gives $D_x = -(A_x + B_x + C_x) = -(0 - [1250\text{ m}]\sin 30.0° + [1000\text{ m}]\cos 40.0°) = -141\text{ m}$.
$A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -(-825\text{ m} + [1250\text{ m}]\cos 30.0° + [1000\text{ m}]\sin 40.0°) = -900\text{ m}$.
The fourth displacement $\vec{D}$ and its components are sketched in Figure 1.83b. $D = \sqrt{D_x^2 + D_y^2} = 911\text{ m}$.

$\tan\phi = \dfrac{|D_x|}{|D_y|} = \dfrac{141\text{ m}}{900\text{ m}}$ and $\phi = 8.9°$. You should head $8.9°$ west of south and must walk 911 m.

(b) The vector diagram is sketched in Figure 1.83c. The final displacement $\vec{D}$ from this diagram agrees with the vector $\vec{D}$ calculated in part (a) using components.

EVALUATE: Note that $\vec{D}$ is the negative of the sum of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

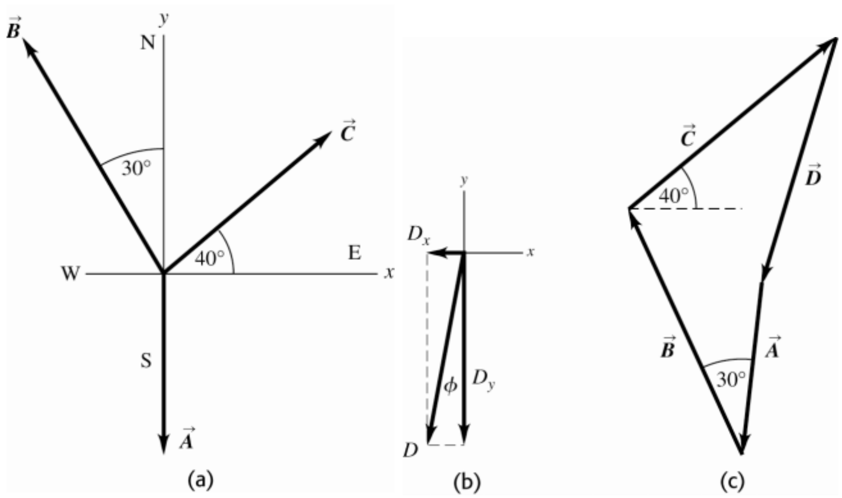


Figure 1.83

1.84. IDENTIFY: If the vector from your tent to Joe's is $\vec{A}$ and from your tent to Karl's is $\vec{B}$, then the vector from Joe's tent to Karl's is $\vec{B}-\vec{A}$.

SET UP: Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

EXECUTE: The position vector for Joe's tent is

$([21.0\text{ m}]\cos 23°)\hat{i} - ([21.0\text{ m}]\sin 23°)\hat{j} = (19.33\text{ m})\hat{i} - (8.205\text{ m})\hat{j}.$

The position vector for Karl's tent is $([32.0\text{ m}]\cos 37°)\hat{i} + ([32.0\text{ m}]\sin 37°)\hat{j} = (25.56\text{ m})\hat{i} + (19.26\text{ m})\hat{j}.$

The difference between the two positions is

$(19.33\text{ m} - 25.56\text{ m})\hat{i} + (-8.205\text{ m} - 19.25\text{ m})\hat{j} = -(6.23\text{ m})\hat{i} - (27.46\text{ m})\hat{j}.$ The magnitude of this vector is the distance between the two tents: $D = \sqrt{(-6.23\text{ m})^2 + (-27.46\text{ m})^2} = 28.2\text{ m}$

EVALUATE: If both tents were due east of yours, the distance between them would be $32.0\text{ m} - 21.0\text{ m} = 17.0\text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0\text{ m} + 21.0\text{ m} = 53.0\text{ m}$. The actual distance between them lies between these limiting values.

1.85. IDENTIFY: In Eqs.(1.21) and (1.27) write the components of $\vec{A}$ and $\vec{B}$ in terms of A, B, θ_A and θ_B.

SET UP: From Appendix B, $\cos(a-b) = \cos a\cos b + \sin a\sin b$ and $\sin(a-b) = \sin a\cos b - \cos a\sin b$.

EXECUTE: **(a)** With $A_z = B_z = 0$, Eq.(1.21) becomes

$A_xB_x + A_yB_y = (A\cos\theta_A)(B\cos\theta_B) + (A\sin\theta_A)(B\sin\theta_B)$

$A_xB_x + A_yB_y = AB(\cos\theta_A\cos\theta_B + \sin\theta_A\sin\theta_B) = AB\cos(\theta_A - \theta_B) = AB\cos\phi$, where the expression for the cosine of the difference between two angles has been used.

(b) With $A_z = B_z = 0$, $\vec{C} = C_z\hat{k}$ and $C = |C_z|$. From Eq.(1.27),

$|C| = |A_xB_y - A_yB_x| = |(A\cos\theta_A)(B\sin\theta_B) - (A\sin\theta_A)(B\cos\theta_A)|$

$|C| = AB|\cos\theta_A\sin\theta_B - \sin\theta_A\cos\theta_B| = AB|\sin(\theta_B - \theta_A)| = AB\sin\phi$, where the expression for the sine of the difference between two angles has been used.

EVALUATE: Since they are equivalent, we may use either Eq.(1.18) or (1.21) for the scalar product and either (1.22) or (1.27) for the vector product, depending on which is the more convenient in a given application.

1.86. IDENTIFY: Apply Eqs.(1.18) and (1.22).

SET UP: The angle between the vectors is $20° + 90° + 30° = 140°$.

EXECUTE: **(a)** Eq. (1.18) gives $\vec{A}\cdot\vec{B} = (3.60\text{ m})(2.40\text{ m})\cos 140° = -6.62\text{ m}^2$.

(b) From Eq.(1.22), the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin 140° = 5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).

EVALUATE: We could also use Eqs.(1.21) and (1.27), with the components of $\vec{A}$ and $\vec{B}$.

1.87. **IDENTIFY:** Compare the magnitude of the cross product, $AB\sin\phi$, to the area of the parallelogram.

SET UP: The two sides of the parallelogram have lengths A and B. ϕ is the angle between $\vec{A}$ and $\vec{B}$.

EXECUTE: **(a)** The length of the base is B and the height of the parallelogram is $A\sin\phi$, so the area is $AB\sin\phi$. This equals the magnitude of the cross product.

(b) The cross product $\vec{A}\times\vec{B}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, so the angle is $90°$.

EVALUATE: It is useful to consider the special cases $\phi = 0°$, where the area is zero, and $\phi = 90°$, where the parallelogram becomes a rectangle and the area is AB.

1.88. **IDENTIFY:** Use Eq.(1.27) for the components of the vector product.

SET UP: Use coordinates with the $+x$-axis to the right, $+y$-axis toward the top of the page, and $+z$-axis out of the page. $A_x = 0$, $A_y = 0$ and $A_z = -3.50\text{ cm}$. The page is 20 cm by 35 cm, so $B_x = 20\text{ cm}$ and $B_y = 35\text{ cm}$.

EXECUTE: $\left(\vec{A}\times\vec{B}\right)_x = 122\text{ cm}^2, \left(\vec{A}\times\vec{B}\right)_y = -70\text{ cm}^2, \left(\vec{A}\times\vec{B}\right)_z = 0.$

EVALUATE: From the components we calculated the magnitude of the vector product is 141 cm^2. $B = 40.3\text{ cm}$ and $\phi = 90°$, so $AB\sin\phi = 141\text{ cm}^2$, which agrees.

1.89. **IDENTIFY:** $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find A, B and the vector difference $\vec{A}-\vec{B}$.

SET UP: $\vec{A} = -2.00\vec{i} + 3.00\vec{j} + 4.00\vec{k},\ \vec{B} = 3.00\vec{i} + 1.00\vec{j} - 3.00\vec{k}$

Use Eq.(1.8) to find the magnitudes of the vectors.

EXECUTE: **(a)** $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

(b) $\vec{A}-\vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A}-\vec{B} = (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + (4.00 - (-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

(c) Let $\vec{C} = \vec{A}-\vec{B}$, so $C_x = -5.00,\ C_y = +2.00,\ C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$, so $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A, B and C are each larger than any of their components.

1.90. **IDENTIFY:** Calculate the scalar product and use Eq.(1.18) to determine ϕ.

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1)+(1)(-1)+(1)(-1) = -1$, so from Eq. (1.18) the angle between the bonds is

$\arccos\left(\dfrac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\dfrac{1}{3}\right) = 109°.$

EVALUATE: The angle between the two vectors in the bond directions is greater than $90°$.

1.91. **IDENTIFY:** Use the relation derived in part (a) of Problem 1.92: $C^2 = A^2 + B^2 + 2AB\cos\phi$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

SET UP: $\cos\phi = 0$ for $\phi = 90°$. $\cos\phi < 0$ for $90° < \phi < 180°$ and $\cos\phi > 0$ for $0° < \phi < 90°$.

EXECUTE: **(a)** If $C^2 = A^2 + B^2$, $\cos\phi = 0$, and the angle between $\vec{A}$ and $\vec{B}$ is $90°$ (the vectors are perpendicular).

(b) If $C^2 < A^2 + B^2$, $\cos\phi < 0$, and the angle between $\vec{A}$ and $\vec{B}$ is greater than $90°$.

(c) If $C^2 > A^2 + B^2$, $\cos\phi > 0$, and the angle between $\vec{A}$ and $\vec{B}$ is less than $90°$.

EVALUATE: It is easy to verify the expression from Problem 1.92 for the special cases $\phi = 0$, where $C = A + B$, and for $\phi = 180°$, where $C = A - B$.

1.92. **IDENTIFY:** Let $\vec{C} = \vec{A} + \vec{B}$ and calculate the scalar product $\vec{C}\cdot\vec{C}$.

SET UP: For any vector $\vec{V}$, $\vec{V}\cdot\vec{V} = V^2$. $\vec{A}\cdot\vec{B} = AB\cos\phi$.

EXECUTE: **(a)** Use the linearity of the dot product to show that the square of the magnitude of the sum $\vec{A}+\vec{B}$ is

$$\left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}+\vec{B}\right) = \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B} = \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + 2\vec{A}\cdot\vec{B} = A^2 + B^2 + 2\vec{A}\cdot\vec{B}$$
$$= A^2 + B^2 + 2AB\cos\phi$$

(b) Using the result of part (a), with $A = B$, the condition is that $A^2 = A^2 + A^2 + 2A^2\cos\phi$, which solves for $1 = 2 + 2\cos\phi$, $\cos\phi = -\frac{1}{2}$, and $\phi = 120°$.

EVALUATE: The expression $C^2 = A^2 + B^2 + 2AB\cos\phi$ is called the law of cosines.

1.93. **IDENTIFY:** Find the angle between specified pairs of vectors.

SET UP: Use $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

EXECUTE: **(a)** $\vec{A} = \hat{k}$ (along line ab)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$A = 1,\; B = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = \hat{k}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = 1/\sqrt{3};\;\; \phi = 54.7°$

(b) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$\vec{B} = \hat{j} + \hat{k}$ (along line ac)

$A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3};\;\; B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}\cdot\vec{B} = \left(\hat{i} + \hat{j} + \hat{k}\right)\cdot\left(\hat{i} + \hat{j}\right) = 1 + 1 = 2$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{2}{\sqrt{3}\sqrt{2}} = \dfrac{2}{\sqrt{6}};\;\; \phi = 35.3°$

EVALUATE: Each angle is computed to be less than $90°$, in agreement with what is deduced from Fig. 1.43 in the textbook.

1.94. **IDENTIFY:** The cross product $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

SET UP: Use Eq.(1.27) to calculate the components of $\vec{A}\times\vec{B}$.

EXECUTE: The cross product is

$(-13.00)\hat{i} + (6.00)\hat{j} + (-11.00)\hat{k} = 13\left[-(1.00)\hat{i} + \left(\dfrac{6.00}{13.00}\right)\hat{j} - \dfrac{11.00}{13.00}\hat{k}\right]$. The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{i} + (6.00/13.00)\hat{j} - (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{i} - (6.00/13.00)\hat{j} + (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{A}$ and $\vec{B}$.

EVALUATE: Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

1.95. **IDENTIFY** and **SET UP:** The target variables are the components of $\vec{C}$. We are given $\vec{A}$ and $\vec{B}$. We also know $\vec{A}\cdot\vec{C}$ and $\vec{B}\cdot\vec{C}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C} = 0$. $A_xC_x + A_yC_y = 0$, which gives $5.0C_x - 6.5C_y = 0$.

$\vec{B}\cdot\vec{C} = 15.0$, so $-3.5C_x + 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = 8.0$ and $C_y = 6.1$

EVALUATE: We can check that our result does give us a vector $\vec{C}$ that satisfies the two equations $\vec{A}\cdot\vec{C} = 0$ and $\vec{B}\cdot\vec{C} = 15.0$.

1.96. **IDENTIFY:** Calculate the magnitude of the vector product and then use Eq.(1.22).

SET UP: The magnitude of a vector is related to its components by Eq.(1.12).

EXECUTE: $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$. $\sin\theta = \dfrac{\left|\vec{A}\times\vec{B}\right|}{AB} = \dfrac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)} = 0.5984$ and

$\theta = \sin^{-1}(0.5984) = 36.8°$.

EVALUATE: We haven't found $\vec{A}$ and $\vec{B}$, just the angle between them.

1.97. **(a) IDENTIFY:** Prove that $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

SET UP: Express the scalar and vector products in terms of components.

EXECUTE:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(\vec{B}\times\vec{C})_x + A_y(\vec{B}\times\vec{C})_y + A_z(\vec{B}\times\vec{C})_z$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x)$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{A}\times\vec{B})_x C_x + (\vec{A}\times\vec{B})_y C_y + (\vec{A}\times\vec{B})_z C_z$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (A_yB_z - A_zB_y)C_x + (A_zB_x - A_xB_z)C_y + (A_xB_y - A_yB_x)C_z$$

Comparison of the expressions for $\vec{A}\cdot(\vec{B}\times\vec{C})$ and $(\vec{A}\times\vec{B})\cdot\vec{C}$ shows they contain the same terms, so

$\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

(b) IDENTIFY: Calculate $(\vec{A}\times\vec{B})\cdot\vec{C}$, given the magnitude and direction of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

SET UP: Use Eq.(1.22) to find the magnitude and direction of $\vec{A}\times\vec{B}$. Then we know the components of $\vec{A}\times\vec{B}$ and of $\vec{C}$ and can use an expression like Eq.(1.21) to find the scalar product in terms of components.

EXECUTE: $A = 5.00$; $\theta_A = 26.0°$; $B = 4.00$, $\theta_B = 63.0°$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$.

The angle ϕ between $\vec{A}$ and $\vec{B}$ is equal to $\phi = \theta_B - \theta_A = 63.0° - 26.0° = 37.0°$. So

$\left|\vec{A}\times\vec{B}\right| = (5.00)(4.00)\sin 37.0° = 12.04$, and by the right hand-rule $\vec{A}\times\vec{B}$ is in the $+z$-direction. Thus

$(\vec{A}\times\vec{B})\cdot\vec{C} = (12.04)(6.00) = 72.2$

EVALUATE: $\vec{A}\times\vec{B}$ is a vector, so taking its scalar product with $\vec{C}$ is a legitimate vector operation. $(\vec{A}\times\vec{B})\cdot\vec{C}$ is a scalar product between two vectors so the result is a scalar.

1.98. **IDENTIFY:** Use the maximum and minimum values of the dimensions to find the maximum and minimum areas and volumes.

SET UP: For a rectangle of width W and length L the area is LW. For a rectangular solid with dimensions L, W and H the volume is LWH.

EXECUTE: **(a)** The maximum and minimum areas are $(L+l)(W+w) = LW + lW + Lw$, $(L-l)(W-w) = LW - lW - Lw$, where the common terms wl have been omitted. The area and its uncertainty are then $WL \pm (lW + Lw)$, so the uncertainty in the area is $a = lW + Lw$.

(b) The fractional uncertainty in the area is $\dfrac{a}{A} = \dfrac{lW + Wl}{WL} = \dfrac{l}{L} + \dfrac{w}{W}$, the sum of the fractional uncertainties in the length and width.

(c) The similar calculation to find the uncertainty v in the volume will involve neglecting the terms lwH, lWh and Lwh as well as lwh; the uncertainty in the volume is $v = lWH + LwH + LWh$, and the fractional uncertainty in the volume is $\dfrac{v}{V} = \dfrac{lWH + LwH + LWh}{LWH} = \dfrac{l}{L} + \dfrac{w}{W} + \dfrac{h}{H}$, the sum of the fractional uncertainties in the length, width and height.

EVALUATE: The calculation assumes the uncertainties are small, so that terms involving products of two or more uncertainties can be neglected.

1.99. **IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.

SET UP: Add the x-components and the y-components.

EXECUTE: The receiver's position is

$[(+1.0+9.0-6.0+12.0)\text{ yd}]\hat{\boldsymbol{i}}+[(-5.0+11.0+4.0+18.0)\text{ yd}]\hat{\boldsymbol{j}}=(16.0\text{ yd})\hat{\boldsymbol{i}}+(28.0\text{ yd})\hat{\boldsymbol{j}}$.

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\text{ yd})\hat{\boldsymbol{i}}+(35.0\text{ yd})\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0\text{ yd})^2+(35.0\text{ yd})^2}=38.5\text{ yd}$. The angle is

$\arctan\left(\dfrac{16.0}{35.0}\right)=24.6°$ to the right of downfield.

EVALUATE: The vector from the quarterback to receiver has positive x-component and positive y-component.

1.100. IDENTIFY: Use the x and y coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

SET UP: If object A has coordinates (x_A, y_A) and object B has coordinates (x_B, y_B), the vector $\vec{r}_{AB}$ from A to B has x-component $x_B - x_A$ and y-component $y_B - y_A$.

EXECUTE: **(a)** The diagram is sketched in Figure 1.100.

(b) (i) In AU, $\sqrt{(0.3182)^2+(0.9329)^2}=0.9857$.

(ii) In AU, $\sqrt{(1.3087)^2+(-0.4423)^2+(-0.0414)^2}=1.3820$.

(iii) In AU $\sqrt{(0.3182-1.3087)^2+(0.9329-(-0.4423))^2+(0.0414)^2}=1.695$.

(c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Equations (1.18) and (1.21),

$$\phi=\arccos\left(\frac{(-0.3182)(1.3087-0.3182)+(-0.9329)(-0.4423-0.9329)+(0)}{(0.9857)(1.695)}\right)=54.6°.$$

(d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

EVALUATE: Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

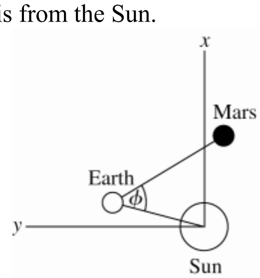


Figure 1.100

1.101. IDENTIFY: Draw the vector addition diagram for the position vectors.

SET UP: Use coordinates in which the Sun to Merak line lies along the x-axis. Let $\vec{A}$ be the position vector of Alkaid relative to the Sun, $\vec{M}$ is the position vector of Merak relative to the Sun, and $\vec{R}$ is the position vector for Alkaid relative to Merak. $A=138\text{ ly}$ and $M=77\text{ ly}$.

EXECUTE: The relative positions are shown in Figure 1.101. $\vec{M}+\vec{R}=\vec{A}$. $A_x=M_x+R_x$ so $R_x=A_x-M_x=(138\text{ ly})\cos 25.6°-77\text{ ly}=47.5\text{ ly}$. $R_y=A_y-M_y=(138\text{ ly})\sin 25.6°-0=59.6\text{ ly}$. $R=76.2\text{ ly}$ is the distance between Alkaid and Merak.

(b) The angle is angle ϕ in Figure 1.101. $\cos\theta=\dfrac{R_x}{R}=\dfrac{47.5\text{ ly}}{76.2\text{ ly}}$ and $\theta=51.4°$. Then $\phi=180°-\theta=129°$.

EVALUATE: The concepts of vector addition and components make these calculations very simple.

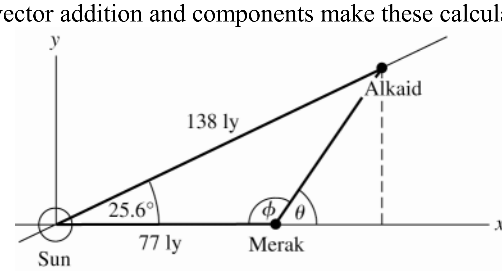


Figure 1.101

1.102. **IDENTIFY:** Define $\vec{S} = A\hat{i} + B\hat{j} + C\hat{k}$. Show that $\vec{r} \cdot \vec{S} = 0$ if $Ax + By + Cz = 0$.

SET UP: Use Eq.(1.21) to calculate the scalar product.

EXECUTE: $\vec{r} \cdot \vec{S} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (A\hat{i} + B\hat{j} + C\hat{k}) = Ax + By + Cz$

If the points satisfy $Ax + By + Cz = 0$, then $\vec{r} \cdot \vec{S} = 0$ and all points $\vec{r}$ are perpendicular to $\vec{S}$. The vector and plane are sketched in Figure 1.102.

EVALUATE: If two vectors are perpendicular their scalar product is zero.

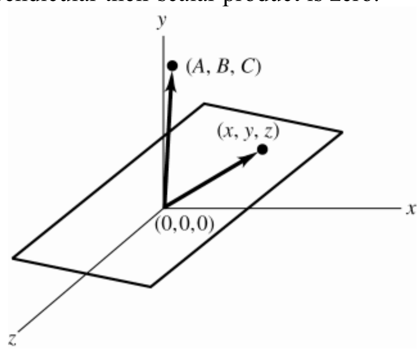


Figure 1.102